Tables of Integral Points on Mordell curves

$$y^2 = x^3 + \kappa$$

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October 3, 2018

At first we consider

The Galois Case

which has not been discussed separately in previous results. We recall that $\Delta = -108\kappa$ must be a square in this case. This is tantamount to $\kappa = -3i^2$ for some natural number i. We therefore order our results with increasing values of i.

In the tables below we list our results in the following form:
In column 1 the value of i is given, column 2 contains the corresponding value of $\kappa = -3 \cdot i^2$. In column 3 we mark points coming from reducible polynomials by "yes". Finally, column 4 contains the points $(x, \pm y)$ for κ. We remark that only i-values are listed for which points exist.

There are three tables. The first one contains the complete results for $1 \leq i \leq 5000$. The entries in the second table contain all points for $100000 \leq i \leq 101000$. We observe that for increasing values of i the occurence of points becomes sparse. Therefore the third table contains results for small intervals of larger i-values only.

We remark that the computation times for κ-values can vary a lot. We therefore only present times for the first two tables. Those calculations were done on a LENOVO TP EDGE with Intel i7 (3rd Generation).

i from 1 to 5000 (2340 seconds CPU time)

i	κ	*Reducible*	*Points*
3	-27	yes	$(3, 0)$
4	-48		$(4, \pm 4),\ (28, \pm 148)$
7	-147		$(91, \pm 868),\ (7, \pm 14)$
9	-243	yes	$(7, \pm 10)$
12	-432		$(12, \pm 36)$
18	-972		$(13, \pm 35)$
24	-1728	yes	$(12, 0)$
26	-2028		$(13, \pm 13)$
30	-2700	yes	$(21, \pm 81)$
32	-3072		$(16, \pm 32),\ (112, \pm 1184)$
39	-4563		$(39, \pm 234)$
42	-5292		$(21, \pm 63)$
45	-6075		$(31, \pm 154),\ (19, \pm 28)$
56	-9408		$(18529, \pm 2522191),\ (364, \pm 6944),\ (49, \pm 329),$ $(28, \pm 112)$
63	-11907	yes	$(43, \pm 260)$
68	-13872		$(1636, \pm 66172),\ (76, \pm 652),\ (52, \pm 356)$
72	-15552	yes	$(28, \pm 80)$
76	-17328		$(28, \pm 68)$
81	-19683	yes	$(27, 0)$
84	-21168	yes	$(1708, \pm 70588),\ (84, \pm 756),\ (28, \pm 28)$ $(57, \pm 405)$
93	-25947		$(31, \pm 62)$
96	-27648		$(48, \pm 288)$
105	-33075	yes	$(39, \pm 162)$
108	-34992	yes	$(252, \pm 3996),\ (36, \pm 108)$ $(73, \pm 595)$
126	-47628	yes	$(37, \pm 55)$
129	-49923		$(5719, \pm 432494),\ (43, \pm 172)$
135	-54675	yes	$(91, \pm 836)$
143	-61347		$(18187, \pm 2452684),\ (247, \pm 3874),\ (91, \pm 832)$
144	-62208	yes	$(52, \pm 280)$
147	-64827		$(147, \pm 1764)$

i	κ	*Reducible*	*Points*
148	-65712		$(196, \pm 2732)$
156	-73008		$(52, \pm 260)$
165	-81675	yes	$(111, \pm 1134)$
180	-97200	yes	$(49, \pm 143)$
186	-103788		$(589, \pm 14291),\ (93, \pm 837)$
189	-107163	yes	$(819, \pm 23436),\ (63, \pm 378)$
		yes	$(67, \pm 440)$
192	-110592	yes	$(48, 0)$
196	-115248		$(49, \pm 49)$
198	-117612	yes	$(133, \pm 1495)$
204	-124848		$(84, \pm 684)$
208	-129792		$(1417, \pm 53339),\ (481, \pm 10543),\ (52, \pm 104)$
212	-134832		$(2092, \pm 95684),\ (52, \pm 76)$
228	-155952		$(5092, \pm 363356),\ (1236, \pm 43452),\ (372, \pm 7164),$ $(228, \pm 3420),\ (156, \pm 1908),\ (76, \pm 532)$
234	-164268	yes	$(157, \pm 1925)$
240	-172800	yes	$(84, \pm 648)$
243	-177147	yes	$(63, \pm 270)$
256	-196608		$(448, \pm 9472),\ (64, \pm 256)$
258	-199692		$(301, \pm 5203)$
270	-218700	yes	$(61, \pm 91)$
273	-223587	yes	$(183, \pm 2430)$
284	-241968		$(604, \pm 14836),\ (148, \pm 1732)$
292	-255792		$(16036, \pm 2030692),\ (892, \pm 26636),\ (76, \pm 428),$ $(73, \pm 365)$
297	-264627	yes	$(103, \pm 910)$
312	-292032		$(156, \pm 1872)$
315	-297675	yes	$(211, \pm 3016),\ (79, \pm 442)$
324	-314928		$(108, \pm 972)$
327	-320787		$(327, \pm 5886)$
336	-338688		$(273, \pm 4473),\ (84, \pm 504)$
360	-388800	yes	$(241, \pm 3689),\ (124, \pm 1232),\ (76, \pm 224)$
365	-399675		$(511, \pm 11534)$
372	-415152		$(217, \pm 3131)$

i	κ	*Reducible*	*Points*
375	-421875	yes	$(75, 0)$
377	-426387		$(91, \pm 572)$
380	-433200		$(12844, \pm 1455628),\ (76, \pm 76)$
381	-435483		$(127, \pm 1270)$
395	-468075		$(79, \pm 158)$
396	-470448	yes	$(97, \pm 665)$
408	-499392	yes	$(273, \pm 4455)$
421	-531723		$(2947, \pm 159980)$
422	-534252		$(1477, \pm 56759)$
429	-552123	yes	$(147, \pm 1620)$
444	-591408		$(86268, \pm 25338132),\ (7428, \pm 640188),$ $(444, \pm 9324),\ (148, \pm 1628),\ (84, \pm 36)$
448	-602112		$(74116, \pm 20177528),\ (1456, \pm 55552),$ $(217, \pm 3101),\ (196, \pm 2632),\ (112, \pm 896)$
459	-632043	yes	$(307, \pm 5320)$
462	-640332	yes	$(93, \pm 405)$
471	-665523		$(1099, \pm 36424)$
474	-674028		$(237, \pm 3555)$
486	-708588	yes	$(117, \pm 945)$
495	-735075	yes	$(91, \pm 136)$
497	-741027	yes	$(91, \pm 112)$
500	-750000		$(700, \pm 18500),\ (100, \pm 500)$
504	-762048	yes	$(172, \pm 2080)$
508	-774192		$(316, \pm 5548)$
513	-789507	yes	$(343, \pm 6290)$
516	-798768		$(129, \pm 1161)$
544	-887808		$(6544, \pm 529376),\ (304, \pm 5216),\ (208, \pm 2848)$
570	-974700	yes	$(381, \pm 7371)$
576	-995328	yes	$(112, \pm 640)$
579	-1005723		$(579, \pm 13896)$
585	-1026675	yes	$(199, \pm 2618),\ (139, \pm 1288)$
588	-1037232		$(196, \pm 2548)$
608	-1108992		$(112, \pm 544)$
620	-1153200		$(9796, \pm 969556),\ (124, \pm 868)$

i	κ	*Reducible*	*Points*
630	-1190700	yes	$(421, \pm 8569),\ (109, \pm 323)$
636	-1213488		$(2604, \pm 132876)$
648	-1259712	yes	$(108, 0)$
652	-1275312		$(27916, \pm 4664228)$
654	-1283148		$(109, \pm 109)$
669	-1342683		$(223, \pm 3122)$
672	-1354752	yes	$(6832, \pm 564704),\ (336, \pm 6048),\ (112, \pm 224),$ $(228, \pm 3240)$
676	-1370928		$(169, \pm 1859)$
693	-1440747	yes	$(463, \pm 9890),\ (163, \pm 1700)$
702	-1478412	yes	$(117, \pm 351),$ $(133, \pm 935)$
716	-1537968		$(244, \pm 3604)$
724	-1572528		$(148, \pm 1292)$
732	-1607472		$(732, \pm 19764)$
734	-1616268		$(1105648789, \pm 36764256872849)$
744	-1660608		$(124, \pm 496)$
759	-1728243	yes	$(507, \pm 11340)$
765	-1755675	yes	$(259, \pm 3952)$
768	-1769472		$(192, \pm 2304)$
780	-1825200	yes	$(5044, \pm 358228),\ (364, \pm 6812),\ (676, \pm 17524),$ $(156, \pm 1404),$ $(129, \pm 567)$
788	-1862832		$(172, \pm 1796)$
796	-1900848		$(124, \pm 76)$
798	-1910412		$(133, \pm 665)$
810	-1968300	yes	$(189, \pm 2187)$
819	-2012283	yes	$(127, \pm 190)$
826	-2046828		$(973, \pm 30317),\ (133, \pm 553)$
828	-2056752	yes	$(553, \pm 12925)$
840	-2116800	yes	$(156, \pm 1296)$
849	-2162403		$(283, \pm 4528)$
852	-2177712		$(516, \pm 11628)$

i	κ	*Reducible*	*Points*
864	-2239488	yes	$(1008, \pm 31968)$, $(144, \pm 864)$, $(292, \pm 4760)$
875	-2296875		$(2275, \pm 108500)$, $(175, \pm 1750)$
889	-2370963		$(679, \pm 17626)$
900	-2430000	yes	$(601, \pm 14651)$
903	-2446227		$(903, \pm 27090)$
906	-2462508		$(453, \pm 9513)$
932	-2605872		$(532, \pm 12164)$
936	-2628288	yes	$(217, \pm 2755)$
938	-2639532		$(301, \pm 4963)$
942	-2662092		$(157, \pm 1099)$
945	-2679075	yes	$(151, \pm 874)$
948	-2696112		$(316, \pm 5372)$
969	-2816883	yes	$(327, \pm 5670)$
975	-2851875	yes	$(651, \pm 16524)$
987	-2922507		$(147, \pm 504)$
990	-2940300	yes	$(181, \pm 1729)$
1004	-3024048		$(268, \pm 4028)$
1008	-3048192	yes	$(148, \pm 440)$
1014	-3084588		$(3549, \pm 211419)$
1029	-3176523	yes	$(147, 0)$
1032	-3195072		$(22876, \pm 459952)$, $(172, \pm 1376)$
1036	-3219888		$(48988, \pm 10842628)$, $(148, \pm 148)$
1053	-3326427	yes	$(351, \pm 6318)$, $(703, \pm 18550)$
1057	-3351747		$(151, \pm 302)$
1068	-3421872		$(156, \pm 612)$
1071	-3441123	yes	$(247, \pm 3410)$
1076	-3473328		$(388, \pm 7412)$
1080	-3499200	yes	$(364, \pm 6688)$

i	κ	*Reducible*	*Points*
1092	-3577392		$(1036, \pm 33292)$, $(532, \pm 12124)$, $(196, \pm 1988)$, $(1092, \pm 36036)$
1095	-3597075		$(219, \pm 2628)$
1125	-3796875	yes	$(175, \pm 1250)$
1134	-3857868	 yes	$(189, \pm 1701)$ $(757, \pm 20735)$
1141	-3905643		$(163, \pm 652)$
1144	-3926208		$(169, \pm 949)$, $(364, \pm 6656)$, $(793, \pm 22243)$, $(988, \pm 30992)$, $(72748, \pm 19621472)$
1152	-3981312	yes	$(208, \pm 2240)$
1164	-4064688		$(388, \pm 7372)$
1176	-4148928		$(588, \pm 14112)$
1184	-4205568		$(784, \pm 21856)$
1197	-4298427	yes	$(403, \pm 7820)$
1204	-4348848		$(172, \pm 860)$
1215	-4428675	yes	$(279, \pm 4158)$, $(171, \pm 756)$
1218	-4450572	yes	$(813, \pm 23085)$
1220	-4465200		$(244, \pm 3172)$
1228	-4523952		$(8596, \pm 796972)$, $(3172, \pm 178636)$, $(436, \pm 8852)$
1248	-4672512		$(208, \pm 2080)$
1260	-4762800	yes	$(169, \pm 253)$
1281	-4922883		$(16023, \pm 2028222)$, $(1659, \pm 67536)$
1281	-4922883		$(427, \pm 8540)$
1284	-4945968		$(228, \pm 2628)$
1292	-5007792		$(5716, \pm 432148)$, $(196, \pm 1588)$, $(172, \pm 284)$
1299	-5062203		$(1299, \pm 46764)$
1305	-5109075	yes	$(871, \pm 25606)$
1313	-5171907		$(9451, \pm 918788)$, $(247, \pm 3146)$
1316	-5195568		$(553, \pm 12803)$
1320	-5227200	yes	$(444, \pm 9072)$, $(201, \pm 1701)$
1326	-5274828	yes	$(237, \pm 2835)$
1368	-5614272	yes	$(313, \pm 5005)$
1372	-5647152		$(196, \pm 1372)$, $(1372, \pm 50764)$

i	κ	*Reducible*	*Points*
1374	-5663628		$(229, \pm 2519)$
1378	-5696652		$(637, \pm 15899)$
1379	-5704923		$(259, \pm 3416)$
1395	-5838075	yes	$(931, \pm 28304)$
1419	-6040683		$(559, \pm 12986)$
1436	-6186288		$(364, \pm 6484)$
1440	-6220800	yes	$(196, \pm 1144)$
1444	-6255408		$(244, \pm 2876)$
1446	-6272748		$(22413, \pm 3355443)$
1449	-6298803	yes	$(487, \pm 10450)$
1477	-6544587		$(211, \pm 1688)$
1482	-6588972		$(741, \pm 20007)$
1488	-6642432	yes	$(2356, \pm 114328)$ $(372, \pm 6696)$, $(217, \pm 1891)$, $(3663921, \pm 7013247273)$, $(993, \pm 31185)$
1500	-6750000		$(300, \pm 4500)$
1512	-6858432	yes	$(166761, \pm 68099157)$, $(3276, \pm 187488)$, $(441, \pm 8883)$, $(252, \pm 3024)$, $(268, \pm 3520)$, $(193, \pm 575)$
1516	-6894768		$(1108, \pm 36788)$
1519	-6922083		$(343, \pm 5782)$
1524	-6967728		$(3937, \pm 247015)$, $(1524, \pm 59436)$, $(804, \pm 22644)$
1530	-7022700	yes	$(349, \pm 5957)$, $(229, \pm 2233)$
1536	-7077888	yes	$(192, 0)$
1544	-7151808		$(193, \pm 193)$
1568	-7375872		$(21217, \pm 3090479)$, $(196, \pm 392)$
1584	-7527168	yes	$(1057, \pm 34255)$, $(532, \pm 11960)$
1596	-7641648		$(988, \pm 30932)$, $(228, \pm 2052)$
1612	-7795632		$(868, \pm 25420)$
1632	-7990272		$(336, \pm 5472)$
1634	-8009868		$(30229, \pm 5255761)$
1659	-8256843		$(399, \pm 7434)$

i	κ	*Reducible*	*Points*
1662	-8286732		$(277, \pm -3601)$
1664	-8306688		$(5668, \pm 426712),\ (1924, \pm 84344),\ (208, \pm 832)$
1668	-8346672		$(556, \pm 12788)$
1683	-8497467	yes	$(1123, \pm 37520),\ (223, \pm 1610)$
1696	-8629248		$(8368, \pm 765472),\ (208, \pm 608)$
1701	-8680203	yes	$(387, \pm 7020)$
1708	-8751792		$(217, \pm 1211)$
1710	-8772300	yes	$(301, \pm 4301)$
1716	-8833968		$(273, \pm 3393)$
1725	-8926875	yes	$(579, \pm 13608)$
1755	-9240075	yes	$(259, \pm 2852)$
1757	-9261147		$(2527, \pm 126994),\ (1267, \pm 44996)$
1767	-9366867		$(1767, \pm 74214)$
1785	-9558675	yes	$(1191, \pm 40986),\ (219, \pm 972)$
1803	-9752427		$(4207, \pm 272854)$
1817	-9904467		$(2923, \pm 158000)$
1824	-9980928		$(4944, \pm 347616),\ (1488, \pm 57312),\ (624, \pm 15264),$ $(304, \pm 4256),\ (20368, \pm 2906848),\ (912, \pm 27360)$
1836	-10112688	yes	$(14724, \pm 1786644),\ (684, \pm 17604),\ (468, \pm 9612)$ $(217, \pm 325)$
1872	-10513152	yes	$(628, \pm 15400)$
1876	-10558128		$(268, \pm 2948)$
1881	-10614483	yes	$(427, \pm 8200)$
1890	-10716300	yes	$(1261, \pm 44659)$
1905	-10887075		$(240919, \pm 118251478)$
1920	-11059200	yes	$(336, \pm 5184)$
1928	-11151552		$(241, \pm 1687)$
1944	-11337408	yes	$(252, \pm 2160)$
1948	-11384112		$(292, \pm 3676)$
1956	-11477808		$(652, \pm 16300),\ (489, \pm 10269),\ (228, \pm 612)$
1995	-11940075	yes	$(215859, \pm 100289448),\ (931, \pm 28196),\ (399, \pm 7182),$ $(291, \pm 3564)$

i	κ	*Reducible*	*Points*
1998	-11976012	yes	$(1333, \pm 48545)$
2012	-12144432		$(3796, \pm 233852)$
2025	-12301875	yes	$(679, \pm 17342)$
2028	-12338352		$(2028, \pm 91260)$
2048	-12582912		$(256, \pm 2048),\ (1792, \pm 75776)$
2052	-12632112		$(252, \pm 1836)$
2064	-12780288		$(1204, \pm 41624)$
2067	-12817467		$(2847, \pm 151866)$
2070	-12854700	yes	$(469, \pm 9503)$
2079	-12966723	yes	$(247, \pm 1450)$
2084	-13029168		$(5332708, \pm 12314639588),\ (1924, \pm 84316)$
2109	-13343643	yes	$(1407, \pm 52650)$
2139	-13725963		$(87823, \pm 26026298),\ (1519, \pm 59086),\ (403, \pm 7192)$
2142	-13764492	yes	$(373, \pm 6175)$
2149	-13854603		$(13699, \pm 1603364),\ (259, \pm 1876)$
2160	-13996800	yes	$(244, \pm 728)$
2164	-14048688		$(4228, \pm 274892)$
2184	-14309568	yes	$(732, \pm 19440)$
2187	-14348907	yes	$(243, 0)$
2196	-14467248		$(133468, \pm 48760228),\ (244, \pm 244)$
2198	-14493612		$(1141, \pm 38353)$
2202	-14546412		$(1101, \pm 36333)$
2212	-14678832		$(3577, \pm 213899),\ (721, \pm 18977)$
2223	-14825187	 yes	$(247, \pm 494)$ $(1483, \pm 56980),\ (283, \pm 2800)$
2250	-15187500	yes	$(325, \pm 4375)$
2268	-15431472	 yes	$(252, \pm 756),\ (15372, \pm 1905876),\ (756, \pm 20412)$ $(513, \pm 10935)$
2272	-15485952		$(2416, \pm 118688),\ (592, \pm 13856)$
2284	-15649968		$(10849, \pm 1130009)$
2307	-15966747		$(2307, \pm 110736)$
2308	-15980592		$(176692, \pm 74272036),\ (1132, \pm 37876)$

i	κ	*Reducible*	*Points*
2331	-16300683		$(9583, \pm 938098),\ (259, \pm 1036)$
2336	-16370688		$(64144, \pm 16245536),\ (3568, \pm 213088),\ (304, \pm 3424),$ $(292, \pm 2920)$
2340	-16426800	yes	$(1561, \pm 61541)$
2349	-16553403	yes	$(787, \pm 21700)$
2364	-16765488		$(2964, \pm 161316)$
2376	-16936128	yes	$(412, \pm 7280)$
2382	-17021772		$(397, \pm 6749)$
2394	-17193708	yes	$(277, \pm 2015)$
2401	-17294403		$(4459, \pm 297724),\ (343, \pm 4802)$
2412	-17453232		$(268, \pm 1340)$
2433	-17758467		$(811, \pm 22708)$
2436	-17802288		$(2548, \pm 128548),\ (364, \pm 5516)$
2452	-18036912		$(412, \pm 7204)$
2460	-18154800	yes	$(1641, \pm 66339)$
2475	-18376875	yes	$(559, \pm 12502)$
2496	-18690048		$(273, \pm 1287),\ (624, \pm 14976)$
2508	-18870192	yes	$(1596, \pm 63612),\ (532, \pm 11476)$ $(273, \pm 1215)$
2511	-18915363		$(279, \pm 1674)$
2516	-18990768		$(988, \pm 30748),\ (508, \pm 10588),\ (268, \pm 508)$
2520	-19051200	yes	$(844, \pm 24128),\ (361, \pm 5291),\ (316, \pm 3536)$
2524	-19111728		$(364, \pm 5396)$
2528	-19172352		$(553, \pm 12245)$
2565	-19737675	yes	$(271, \pm 406)$
2580	-19969200		$(516, \pm 10836)$
2583	-20015667	yes	$(1723, \pm 71380)$
2592	-20155392		$(432, \pm 7776)$
2604	-20342448		$(4809, \pm 333459),\ (2604, \pm 132804),\ (372, \pm 5580)$ $(273, \pm 63)$
2608	-20404992		$(3097, \pm 172291)$
2616	-20530368		$(1308, \pm 47088)$

i	κ	*Reducible*	*Points*
2622	-20624652	yes	$(453, \pm 8505)$
2628	-20719152		$(292, \pm 2044)$
2653	-21115227		$(511, \pm 10598)$
2688	-21676032		$(1092, \pm 35784),\ (336, \pm 4032)$
2691	-21724443	yes	$(607, \pm 14210)$
2697	-21821427	yes	$(18507, \pm 2517696),\ (1147, \pm 38564)$ $(903, \pm 26730)$
2702	-21902412		$(11557, \pm 1242409),\ (301, \pm 2317)$
2709	-22016043	yes	$(1807, \pm 76670)$
2730	-22358700	yes	$(309, \pm 2673)$
2748	-22654512		$(2977, \pm 162361)$
2763	-22902507		$(307, \pm 2456)$
2796	-23452848		$(43068, \pm 8937828)$
2804	-23587248		$(17404, \pm 2296004),\ (364, \pm 4964)$
2805	-23604075	yes	$(399, \pm 6318)$
2812	-23722032		$(1015612, \pm 1023509164),\ (916, \pm 27292),\ (556, \pm 12172),$ $(292, \pm 1084)$
2814	-23755788		$(469, \pm 8911)$
2835	-24111675	yes	$(351, \pm 4374)$
2838	-24162732	yes	$(3397, \pm 197929),\ (301, \pm 1763),\ (1677, \pm 68499),$ $(1893, \pm 82215)$
2847	-24316227		$(507, \pm -10296)$
2880	-24883200	yes	$(964, \pm 29512),\ (496, \pm 9856),\ (304, \pm 1792)$
2884	-24952368		$(7828, \pm 692572)$
2891	-25073643		$(6223, \pm 490882)$
2892	-25090992		$(6748, \pm 554300)$
2899	-25212603		$(1183, \pm 40378)$
2916	-25509168	yes	$(324, \pm 2916),\ (2268, \pm 107892),$ $(657, \pm 16065)$
2919	-25561683		$(2919, \pm 157626)$
2920	-25579200		$(2044, \pm 92272)$
2956	-26213808		$(724, \pm 18796)$

i	κ	*Reducible*	*Points*
2970	-26462700	yes	$(1981, \pm 88021),\ (301, \pm 899)$
2976	-26569728		$(868, \pm 25048)$
2982	-26676972		$(9597, \pm 940149),\ (2037, \pm 91791)$
3000	-27000000	yes	$(300, 0)$
3010	-27180300		$(301, \pm 301)$
3012	-27216432		$(4476, \pm 299412)$
3016	-27288768		$(364, \pm 4576)$
3028	-27506352		$(316, \pm 2012)$
3031	-27560883		$(7819, \pm 691376)$
3040	-27724800		$(51376, \pm 11645024),\ (304, \pm 608)$
3048	-27870912		$(508, \pm 10160)$
3052	-27944112		$(436, \pm 7412)$
3066	-28201068		$(1533, \pm 59787)$
3068	-28237872		$(55341481, \pm 411695543363),\ (481, \pm 9113)$
3069	-28256283	yes	$(1027, \pm 32480)$
3087	-28588707	yes	$(343, \pm 3430)$
3105	-28923075	yes	$(2071, \pm 94094),\ (439, \pm 7462)$
3108	-28978992		$(777, \pm 20979)$
3116	-29128368		$(532, \pm 11020)$
3143	-29635347		$(427, \pm 6944)$
3150	-29767500	yes	$(709, \pm 18073),\ (541, \pm 11339)$
3153	-29824227		$(1051, \pm 33632)$
3160	-29956800		$(316, \pm 1264)$
3168	-30108672	yes	$(388, \pm 5320)$
3207	-30854547		$(276871, \pm 145685458)$
3236	-31415088		$(9364, \pm 906116)$
3243	-31551147	yes	$(2163, \pm 100440)$
3244	-31570608		$(1348, \pm 49172)$
3250	-31687500		$(325, \pm 1625)$
3252	-31726512		$(3252, \pm 185364),\ (876, \pm 25308)$
3264	-31961088	yes	$(1092, \pm 35640)$
3276	-32196528	yes	$(2548, \pm 128492),\ (364, \pm 4004)$ $(337, \pm 2465)$

i	κ	*Reducible*	*Points*
3283	-32334267		$(931, \pm 27832)$
3297	-32610627		$(471, \pm 8478)$
3308	-32828592		$(1516, \pm 58748)$
3368	-34030272		$(11788, \pm 1279840)$
3376	-34192128		$(5908, \pm 454072)$
3384	-34354368	yes	$(2257, \pm 107065)$
3393	-34537347	yes	$(763, 20240)$
3402	-34720812	yes	$(333, \pm 1485)$
3419	-35068683		$(403, \pm 5512)$
3420	-35089200	yes	$(481, \pm 8729)$
3432	-35335872	yes	$(588, \pm 12960)$
3444	-35583408		$(364, \pm 3556)$
3465	-36018675	yes	$(1159, \pm 38998),\ (379, \pm 4292),\ (331, \pm 496)$
3468	-36081072		$(372, \pm 3924)$
3472	-36164352		$(1876, \pm 81032),\ (532, \pm 10696)$
3483	-36393867		$(51471, \pm 11677338),\ (387, \pm 4644)$
3488	-36498432		$(496, \pm 9248)$
3490	-36540300		$(349, \pm 2443)$
3519	-37150083	yes	$(427, \pm 6380)$
3521	-37192323		$(343, \pm 1778)$
3528	-37340352	yes	$(2353, \pm 113975)$
3532	-37425072		$(45292, \pm 9639004)$
3552	-37850112		$(345072, \pm 202705056),\ (29712, \pm 5121504),$ $(1776, \pm 74592),\ (592, \pm 13024),\ (336, \pm 288)$
3584	-38535168		$(296464, \pm 161420224),\ (5824, \pm 444416),$ $(868, \pm 24808),\ (784, \pm 21056),\ (448, \pm 7168)$
3603	-38944827		$(3603, \pm 216180)$
3604	-38966448		$(13468, \pm 1562972)$
3612	-39139632		$(532, \pm 10556)$
3633	-39596067		$(24087, \pm 3738294),\ (651, \pm 15372)$
3645	-39858075	yes	$(819, \pm 22572)$
3650	-39967500		$(949, \pm 28543)$

i	κ	*Reducible*	*Points*
3660	-40186800		$(5124, \pm 366732)$
3672	-40450752	yes	$(1228, \pm 42560)$
3675	-40516875	yes	$(2451, \pm 121176)$
3696	-40981248	yes	$(372, \pm 3240)$
3708	-41247792		$(412, \pm 5356)$
3726	-41649228	yes	$(637, \pm 14725)$
3748	-42142512		$(772, \pm 20444)$
3750	-42187500	yes	$(525, \pm 10125)$
3756	-42322608		$(1252, \pm 43820)$
3768	-42593472		$(4396, \pm 291392)$
3792	-43137792		$(948, \pm 28440)$
3810	-43548300		$(381, \pm 3429)$
3812	-43594032		$(436, \pm 6268)$
3825	-43891875	yes	$(2551, \pm 128674)$
3829	-43983723		$(547, \pm 10940)$
3861	-44721963	yes	$(163683, \pm 66222468)$, $(2223, \pm 104598)$, $(819, \pm 22464)$, $(367, \pm 2170)$
3864	-44791488	yes	$(417, \pm 5265)$
3876	-45070128		$(444, \pm 6516)$
3884	-45256368		$(364, \pm 1724)$
3885	-45279675	yes	$(1299, \pm 46332)$
3888	-45349632	yes	$(468, \pm 7560)$
3900	-45630000		$(364, \pm 1612)$
3906	-45770508	yes	$(877, \pm 25075)$
3950	-46807500		$(1501, \pm 57749)$
3951	-46831203		$(439, \pm 6146)$
3960	-47044800	yes	$(364, \pm 1088)$
3962	-47092332		$(637, \pm 14539)$
3969	-47258883		$(1323, \pm 47628)$
3972	-47330352		$(3972, \pm 250236)$
3976	-47425728		$(364, \pm 896)$
3978	-47473452	yes	$(2653, \pm 136475)$
3993	-47832147	yes	$(363, 0)$

i	κ	*Reducible*	*Points*
3996	-47904048		$(1764, \pm 73764)$
4000	-48000000		$(400, \pm 4000)$, $(2800, \pm 148000)$
4004	-48096048		$(297388, \pm 162175468)$, $(364, \pm 364)$
4011	-48264363		$(399, \pm 3906)$
4020	-48481200		$(804, \pm 21708)$
4028	-48674352		$(676, \pm 16132)$, $(652, \pm 15116)$, $(1693876, \pm 2204562532)$
4032	-48771072	yes	$(688, \pm 16640)$
4036	-48867888		$(509884, \pm 364088596)$, $(508, \pm 9068)$
4037	-48892107		$(367, \pm 734)$
4064	-49548288		$(1264, \pm 44384)$
4074	-49792428		$(2037, \pm 91665)$
4095	-50307075	yes	$(571, \pm 11656)$
4097	-50356227		$(1687, \pm 68926)$
4104	-50528448	yes	$(1372, \pm 50320)$
4116	-50824368		$(588, \pm 12348)$
4128	-51121152		$(516, \pm 9288)$, $(817, \pm 22231)$
4134	-51269868	yes	$(2757, \pm 144585)$
4140	-51418800	yes	$(409, \pm 4123)$
4169	-52141683		$(379, \pm 1516)$
4176	-52316928	yes	$(937, \pm 27755)$
4188	-52618032		$(1396, \pm 51652)$
4210	-53172300		$(421, \pm 4631)$
4212	-53222832		$(468, \pm 7020)$
4244	-54034608		$(388, \pm 2092)$
4252	-54238512		$(5212, \pm 376204)$
4268	-54647472		$(388, \pm 1940)$
4275	-54826875	yes	$(511, \pm 8866)$
4284	-55057968	yes	$(457, \pm 6355)$
4293	-55289547	yes	$(2863, \pm 153010)$
4300	-55470000		$(1204, \pm 41108)$
4329	-56220723	yes	$(1447, \pm 54530)$
4347	-56689227	yes	$(403, \pm 2960)$

i	κ	*Reducible*	*Points*
4350	-56767500	yes	$(741, \pm 18711)$
4352	-56819712		$(26176, \pm 4235008),\ (1216, \pm 41728),\ (832, \pm 22784)$
4359	-57002643		$(4359, \pm 287694)$
4371	-57316923		$(25699, \pm 4119776),\ (3999, \pm 252774),\ (403, \pm 2852)$
4388	-57763632		$(1036, \pm 32468)$
4398	-58027212		$(733, \pm 18325)$
4413	-58423707		$(1471, \pm 55898)$
4417	-58529667		$(427, \pm 4396)$
4455	-59541075	yes	$(2971, \pm 161756),\ (999, \pm 30618),\ (619, \pm 13328)$
4476	-60103728		$(28348, \pm 4772908)$
4484	-60318768		$(532, \pm 9500)$
4485	-60345675	yes	$(399, \pm 1782)$
4491	-60507243		$(499, \pm 7984)$
4524	-61399728		$(1209, \pm 41301)$
4532	-61617072		$(412, \pm 2884)$
4548	-62052912		$(19708, \pm -2766700),\ (1956, \pm 86148),\ (1137, \pm 37521)$
4554	-62216748	yes	$(397, \pm 595)$
4560	-62380800	yes	$(1524, \pm 58968)$
4589	-63176763		$(403, \pm 1508)$
4602	-63535212		$(29757, \pm 5133141),\ (1677, \pm 68211)$
4606	-63645708		$(2989, \pm 163219),\ (1813, \pm 76783)$
4608	-63700992	yes	$(448, \pm 5120)$
4620	-64033200	yes	$(3081, \pm 170829)$
4632	-64366272		$(2316, \pm 111168)$
4636	-64477488		$(88084, \pm 26142404)$
4680	-65707200	yes	$(796, \pm 20944),\ (556, \pm 10304)$
4704	-66382848		$(784, \pm 20384)$
4706	-66439308		$(2509, \pm 125411)$
4725	-66976875	yes	$(499, \pm 7568)$
4740	-67402800		$(31521, \pm 5596281)$
4743	-67488147	yes	$(1063, \pm 33670)$

i	κ	*Reducible*	*Points*
4748	-67630512		$(2236, \pm 105412)$
4764	-68087088		$(4764, \pm 328716),\ (588, \pm 11628)$
4771	-68287323		$(6643, \pm 541372)$
4788	-68774832	yes	$(3193, \pm 180235)$
4797	-69033627	yes	$(1603, \pm 63640)$
4828	-69928752		$(412, \pm 76)$
4830	-69986700	yes	$(669, \pm 15147)$
4836	-70160688		$(1092, \pm 35100)$
4860	-70858800	yes	$(441, \pm 3861)$
4864	-70975488		$(448, \pm 4352)$
4875	-71296875		$(975, \pm 29250)$
4881	-71472483		$(1627, \pm 65080)$
4884	-71560368		$(46324, \pm 9970316),\ (4588, \pm 310652),\ (444, \pm 3996)$
4956	-73685808		$(2793, \pm 147357)$
4959	-73775043	yes	$(3307, \pm 189980)$
4960	-73804800		$(39184, \pm 7756448),\ (496, \pm 6944)$
4988	-74640432		$(3268, \pm 186620)$

i from 100000 to 101000 (2500 seconds CPU time)

i	κ	*Reducible*	*Points*
100005	-30003000075	yes	$(13359, \pm 1534302)$
100035	-30021003675	yes	$(3199, \pm 52118)$
100074	-30044416428		$(11973, \pm 1298583)$
100086	-30051622188		$(7149, \pm 579069)$
100104	-30062432448		$(7009, \pm 560591)$, $(8428, \pm 754048)$, $(12513, \pm 1388943)$
100116	-30069640368		$(3708, \pm 144612)$
100148	-30088865712		$(6076, \pm 440708)$
100156	-30093673008		$(3577, \pm 125195)$, $(3724, \pm 146804)$, $(43708, \pm 9136148)$, $(785764, \pm 696527356)$
100233	-30139962867	yes	$(3367, \pm 89614)$, $(28231, \pm 4740218)$, $(66823, \pm 17272970)$
100236	-30141767088		$(8353, \pm 743417)$, $(33348, \pm 6087348)$, $(7450788, \pm 20337768972)$
100352	-30211571712		$(3136, \pm 25088)$, $(339472, \pm 197790656)$
100389	-30233853963		$(702723, \pm 589082652)$
100464	-30279045888	yes	$(33492, \pm 6126840)$
100485	-30291705675	yes	$(6799, \pm 532918)$
100533	-30320652267		$(9331, \pm 884368)$
100564	-30339354288		$(3244, \pm 61636)$
100566	-30340561068	yes	$(22357, \pm 3338335)$
100594	-30357458508		$(3133, \pm 19877)$, $(5629, \pm 384709)$
100602	-30362287212	yes	$(5733, \pm 397575)$
100674	-30405762828	yes	$(3637, \pm 133055)$
100725	-30436576875		$(10191, \pm 1013886)$
100728	-30438389952		$(5596, \pm 380528)$
100732	-30440807472		$(160852, \pm 64511644)$
100737	-30443829507	yes	$(7543, \pm 631450)$
100768	-30462569472		$(13936, \pm 1635872)$
100802	-30483129612		$(3877, \pm 166711)$
100852	-30513377712		$(32908, \pm 5967140)$
100884	-30532744368		$(4804, \pm 283436)$, $(7364532, \pm 19985624820)$
100896	-30540008448		$(3153, \pm 28377)$
100939	-30566045163		$(157423, \pm 62459798)$
100980	-30590881200	yes	$(4249, \pm 214757)$

Several larger values i for which points exist

i	κ	*Reducible*	*Points*
500005	-750015000075		$(9091, \pm 36364)$
500052	-750156008112		$(260841, \pm 133215453),\ (71436, \pm 19073412)$
1000008	-3000048000192	yes	$(666673, \pm 544336055)$
1000128	-3000768049152		$(20836, \pm 2458648)$
1000146	-3000876063948		$(28749, \pm 4556349)$
1000188	-3001128106032	yes	$(15876, \pm 1000188),\ (111132, \pm 37006956)$ $(32193, \pm 5510295)$
1000377	-3002262426387		$(14427, \pm 23436)$
1000395	-3002370468075	yes	$(222319, \pm 104810678),\ (15151, \pm 689626)$
1000605	-3003631098075		$(21151, \pm 2541374)$
35000005	-3675001050000075		$(18583759, \pm 80112488602)$

From now on we consider

The non-Galois case

which is definitely more frequent.

Extension of special examples of Wildanger and Jätzschmann

K. Wildanger (in his thesis) and later A. Jätzschmann (in his Diplomarbeit) calculated points for Mordell curves with special values of $\pm\kappa$. Wildanger achieved absolut values of κ up to 10^7 and Jätzschmann up to 10^9. Making use of methods from class field theory we could extend this range considerably.

Remarks

1. Before we present a table with several of our results we would like to point out that Mordell curves with $\kappa = 10^{3i} + 25$ and $i \in \mathbb{N}$ have points with comparatively large coordinates. From the results of Jätzschmann we observe that there seems to exist a point with coordinates

   $$\left(2 \cdot 10^i + 9 \cdot 10^{4i-2},\ \pm(5 + 9 \cdot 10^{3i-1} + 27 \cdot 10^{3(2i-1)})\right) .$$

   This is easily verified by a straightforward calculation.
   Another family of points is obviously

   $$\kappa = 10^{3i} + 25,\ x = -10^i,\ y = \pm 5 .$$

2. In the following examples there are almost no points coming from reducible polynomials. The only exceptions occur in the next two tables.

κ	*Reducible*	*Points*
1000000000025		$(-10000, \pm 5),\ (900000000020000, \pm 27000000000900000000005)$
100000000025		$\emptyset$
10000000025		$(-1570, \pm 78295),\ (-100, \pm 99995),\ (100, \pm 100005),$ $(1000000, \pm 1000000005)$
1000000025		$(-1000, \pm 5),\ (-170, \pm 31545),\ (1271, \pm 55256),$ $(2614, \pm 137337),\ (90000002000, \pm 27000000900000005)$
100000025		$\emptyset$
10000025		$(10000, \pm 1000005)$
1000025		$(-100, \pm 5),\ (-64, \pm 859),\ (55, \pm 1080),\ (94, \pm 1353),$ $(166, \pm 2361),\ (740, \pm 20155),\ (9000200, \pm 27000900005)$
100025		$(40, \pm 405)$
10025		$(-20, \pm 45),\ (-16, \pm 77),\ (-10, \pm 95),\ (10, \pm 105),\ (14, \pm 113),$ $(35, \pm 230),\ (55, \pm 420),\ (100, \pm 1005),\ (226, \pm 3399)$
1025		$(-10, \pm 5),\ (-5, \pm 30),\ (-4, \pm 31),\ (-1, \pm 32),\ (4, \pm 33),$ $(10, \pm 45),\ (20, \pm 95),\ (40, \pm 255),\ (50, \pm 355),$ $(64, \pm 513),\ (155, \pm 1930),\ (166, \pm 2139),\ (446, \pm 9419),$ $(920, \pm 27905),\ (3631, \pm 218796),\ (3730, \pm 227805)$
125	yes	$(-5, 0)$
-125	yes	$(5, 0)$
-1025		$\emptyset$
-10025		$\emptyset$
-100025		$(69, \pm 478)$
-1000025		$(101, \pm 174)$
-10000025		$(249, \pm 2332)$
-100000025		$\emptyset$
-1000000025		$\emptyset$
-10000000025		$\emptyset$
-100000000025		$\emptyset$
-1000000000025		$\emptyset$

Several new large examples

κ	*Reducible*	*Points*
10^{12}		$(0, \pm 10^6)$, $(20000, \pm 3 * 10^6)$
	yes	$(-10000, 0)$
-10^{12}	yes	$(10000, 0)$
10^{15}		$(650000, \pm 525 * 10^6)$, $(-60000, \pm 28 * 10^6)$
	yes	$(-100000, 0)$
-10^{15}	yes	$(100000, 0)$
10^{18}		$(0, \pm 10^9)$, $(2 * 10^6, \pm 3 * 10^9)$
	yes	$(-1000000, 0)$
-10^{18}	yes	$(1000000, 0)$
10^{20}		$\emptyset$
-10^{20}	yes	$(34000000, \pm 198000000000)$
15^{10}		$(-8100, \pm 212625)$, $(0, \pm 759375)$, $(22500, \pm 3459375)$
15^{12}		$(0, \pm 11390625)$, $(101250), \pm 34171875)$
	yes	$(-50625, 0)$
-15^{12}	yes	$(50625, 0)$
15^{15}	yes	$(-759375, 0)$
-15^{15}	yes	$(759375, 0)$
42^{10}		$(0, \pm 130691232)$
	yes	$(-252252, \pm 32080104)$
-42^{10}		$\emptyset$
42^{12}	yes	$(-3111696, 0)$
-42^{12}	yes	$(3111696, 0)$
60^8		$(-54000, \pm 3240000)$, $(0, \pm 12960000)$, $(86400, \pm 28512000)$
		$(144000, \pm 56160000)$, $(1930500, \pm 2682315000)$
-60^8		$(522000, \pm 376920000)$
60^9	yes	$(-216000, 0)$
-60^9	yes	$(216000, 0)$
60^{10}		$(0, \pm 777600000)$, $(1120000, \pm 1417600000)$, $(2721600, \pm 4556736000)$
-60^{10}		$\emptyset$

We observe that a point $(x, \pm y)$ for κ corresponds to a point $(100x, \pm 1000y)$ for $10^6\kappa$.

On the other hand, for

$$\kappa \in \left\{ \begin{array}{l} \pm 5^2 * 7^2 * 11 * 13 * 17 * 19^2, \\ \pm 5^2 * 7^2 * 11 * 13 * 17 * 19^2 * 23 * 29, \\ -5 * 7 * 11 * 13 * 17 * 19 * 23 * 29 * 101 * 103 \end{array} \right\}$$

there are no points.

Curves with many points

We also calculated tables of curves with many points. From those we only present those which do not occur in the tables of M. A. Bennett and A. Ghadermarzi:
$http://www.math.ubc.ca/ \sim bennett/BeGa - data.html$).
The last four κ–values were chosen from the tables of D. J. Broadhurst
($http://physics.open.ac.uk/ \sim dbroadhu/cert/mwrank8$ and
$http://physics.open.ac.uk/ \sim dbroadhu/cert/mwrank9$).

32 points for $k = 1000016$
$(-100, \pm 4),\ (-68, \pm 828),\ (-31, \pm 985),\ (-20, \pm 996),\ (20, \pm 1004),$ $(52, \pm 1068),\ (65, \pm 1129),\ (97, \pm 1383),\ (580, \pm 14004),\ (844, \pm 24540),$ $(1532, \pm 59972),\ (2500, \pm 125004),\ (19892, \pm 2805548),\ (28444, \pm 4797180),$ $(741985, \pm 639135129),\ (14062700, \pm 52735500004)$
30 points for $k = 1023849$
$(-90, \pm 543),\ (-24, \pm 1005),\ (-12, \pm 1011),\ (40, \pm 1043),\ (138, \pm 1911),$ $(163, \pm 2314),\ (210, \pm 3207),\ (226, \pm 3545),\ (375, \pm 7332),\ (1008, \pm 32019),$ $(3480, \pm 205293),\ (4728, \pm 325101),\ (8260, \pm 750707),\ (28260, \pm 4750707),$ $(177648, \pm 74875629)$
28 points for $k = 1037025$
$(-101, \pm 82),\ (-90, \pm 555),\ (-50, \pm 955),\ (-30, \pm 1005),\ (15, \pm 1020),$ $(36, \pm 1041),\ (114, \pm 1587),\ (240, \pm 3855),\ (300, \pm 5295),\ (796, \pm 22481),$ $(955, \pm 29530),\ (2040, \pm 92145),\ (9684, \pm 952977),\ (337101810, \pm 6189301322445)$
32 points for $k = 1072953$
$(-84, \pm 693),\ (-48, \pm 981),\ (-38, \pm 1009),\ (-12, \pm 1035),\ (7, \pm 1036),$ $(42, \pm 1071),\ (51, \pm 1098),\ (196, \pm 2933),\ (238, \pm 3815),\ (438, \pm 9225),$ $(672, \pm 17451),\ (1491, \pm 57582),\ (3196, \pm 180683),\ (11886, \pm 1295847),$ $(39687, \pm 7906284),\ (57988, \pm 13963915)$

34 points for $k = 1126233$
$(-104, \pm 37),\ (-102, \pm 255),\ (-68, \pm 901),\ (-8, \pm 1061),\ (36, \pm 1083),$ $(51, \pm 1122),\ (102, \pm 1479),\ (132, \pm 1851),\ (162, \pm 2319),\ (306, \pm 5457),$ $(531, \pm 12282),\ (748, \pm 20485),\ (1326, \pm 48297),\ (2448, \pm 121125),$ $(12087, \pm 1328856),\ (19027, \pm 2624554),\ (21522, \pm 3157359)$
28 points for $k = 1145313$
$(-104, \pm 143),\ (-78, \pm 819),\ (-42, \pm 1035),\ (12, \pm 1071),\ (66, \pm 1197),$ $(91, \pm 1378),\ (156, \pm 2223),\ (208, \pm 3185),\ (507, \pm 11466),\ (858, \pm 25155),$ $(1222, \pm 42731),\ (1551, \pm 61092),\ (1686, \pm 69237),\ (10452, \pm 1068561)$
40 points for $k = 1145889$
$(-104, \pm 145),\ (-102, \pm 291),\ (-72, \pm 879),\ (30, \pm 1083),\ (46, \pm 1115),$ $(60, \pm 1167),\ (96, \pm 1425),\ (108, \pm 1551),\ (123, \pm 1734),\ (1200, \pm 41583),$ $(1318, \pm 47861),\ (1695, \pm 69792),\ (4588, \pm 310769),\ (5028, \pm 356529),$ $(5535, \pm 411792),\ (13470, \pm 1563333),\ (26463, \pm 4304856),\ (47730, \pm 10427667),$ $(61300, \pm 15177167),\ (572046, \pm 432659685)$
32 points for $k = 1157264$
$(-104, \pm 180),\ (-79, \pm 815),\ (8, \pm 1076),\ (25, \pm 1083),\ (80, \pm 1292),$ $(88, \pm 1356),\ (128, \pm 1804),\ (160, \pm 2292),\ (193, \pm 2889),\ (440, \pm 9292),$ $(1096, \pm 36300),\ (6025, \pm 467667),\ (16096, \pm 2042100),\ (24296, \pm 3787060),$ $(48521, \pm 10687955),\ (109393, \pm 36181311)$

26 points for $k = 1000012401$
$(-990, \pm 5451),\ (-876, \pm 18105),\ (-828, \pm 20793),\ (-510, \pm 29451),$ $(12, \pm 31623),\ (1012, \pm 45127),\ (1899, \pm 88590),\ (2347, \pm 118018),$ $(4000, \pm 254951),\ (4840, \pm 338201),\ (14187, \pm 1690098),\ (26302, \pm 4265747),$ $(28062, \pm 4700973)$
36 points for $k = 1000161604$
$(-1000, \pm 402),\ (-120, \pm 31598),\ (116, \pm 31650),\ (260, \pm 31902),$ $(780, \pm 38402),\ (1040, \pm 46098),\ (1365, \pm 59527),\ (1668, \pm 75106),$ $(3653, \pm 223041),\ (3848, \pm 240786),\ (5916, \pm 456130),\ (8700, \pm 812098),$ $(34541, \pm 6419595),\ (71828, \pm 19250466),\ (103220, \pm 33162402),$ $(103805, \pm 33444723),\ (1168493, \pm 1263104181),\ (3930488, \pm 7792372626)$
46 points for $k = 2000651625$
$(-1260, \pm 525),\ (-1134, \pm 23289),\ (-656, \pm 41453),\ (-594, \pm 42321),$ $(-276, \pm 44493),\ (-210, \pm 44625),\ (126, \pm 44751),\ (490, \pm 46025),$ $(840, \pm 50925),\ (996, \pm 54669),\ (1446, \pm 70881),\ (3234, \pm 189273),$ $(3319, \pm 196372),\ (3924, \pm 249843),\ (5896, \pm 454931),\ (7651, \pm 670726),$ $(16990, \pm 2215025),\ (38115, \pm 7441350),\ (49434, \pm 10991127),\ (70840, \pm 18854675),$ $(75399, \pm 20703768),\ (20577816, \pm 93346689261),\ (75144090, \pm 651391735575)$
26 points for $k = 4000250025$
$(-1500, \pm 25005),\ (76, \pm 63251),\ (570, \pm 64695),\ (1330, \pm 79705),$ $(1539, \pm 87438),\ (3895, \pm 251180),\ (4560, \pm 314355),\ (8994, \pm 855303),$ $(11416, \pm 1221389),\ (11590, \pm 1249345),\ (22540, \pm 3384595),\ (34200, \pm 6325005),$ $(66435, \pm 17123730)$

40 points for $k = 8000032400$
$(-2000, \pm 180),\ (-1480, \pm 68980),\ (-1304, \pm 76044),\ (-160, \pm 89420),$ $(424, \pm 89868),\ (944, \pm 94028)\ (1160, \pm 97780),\ (1801, \pm 117651),$ $(2080, \pm 130380),\ (2536, \pm 155916),\ (2945, \pm 183145),\ (5905, \pm 462495),$ $(15185, \pm 1873345),\ (16040, \pm 2033420),\ (21169, \pm 3081297),\ (58264, \pm 14064012),$ $(93985, \pm 28813095),\ (681760, \pm 562920780),\ (786440, \pm 697426420),$ $(143493400, \pm 1718889224820)$
78 points for $k = 1632201497$
$(-1177, \pm 1292),\ (-1132, \pm 13477),\ (-1088, \pm 18555),\ (-982, \pm 26177),$ $(-656, \pm 36741),\ (-557, \pm 38202),\ (-538, \pm 38425),\ (34, \pm 40401),$ $(844, \pm 47259),\ (1079, \pm 53744),\ (1144, \pm 55941),\ (2708, \pm 146597),$ $(2948, \pm 165083),\ (3014, \pm 170329),\ (3434, \pm 205249),\ (3538, \pm 214287),$ $(7162, \pm 607455),\ (7264, \pm 620421),\ (7304, \pm 625531),\ (10483, \pm 1074078),$ $(11458, \pm 1227153),\ (18128, \pm 2441093),\ (18212, \pm 2458075),\ (36928, \pm 7096443),$ $(38887, \pm 7668540),\ (51994, \pm 11855841),\ (116348, \pm 3968611),\ (180154, \pm 76465569),$ $(406963, \pm 259616562),\ (485764, \pm 338561829),\ (532204, \pm 388255131),\ (574262, \pm 435176185),$ $(25820762, \pm 131205965665),\ (2670928, \pm 4365090693),\ (16266184, \pm 65603728251),$ $(56730554, \pm 427292762719),\ (75380023, \pm 654461944608),\ (144899018, \pm 1744207555073),$ $(2756665418, \pm 144735840400127)$

100 points for $k = -24864246775$
$(2930, \pm 17015)$, $(3011, \pm 49334)$, $(3040, \pm 56835)$, $(3134, \pm 76927)$, $(3610, \pm 148935)$, $(3634, \pm 152073)$, $(3770, \pm 169465)$, $(4420, \pm 247965)$, $(4474, \pm 254343)$, $(6544, \pm 505347)$, $(6800, \pm 538115)$, $(6830, \pm 541985)$, $(9194, \pm 867353)$, $(10135, \pm 1008060)$, $(10580, \pm 1076765)$, $(15094, \pm 1847697)$, $(16675, \pm 2147490)$, $(19139, \pm 2643062)$, $(20320, \pm 2892285)$, $(31096, \pm 5481219)$, $(32470, \pm 5848785)$, $(35120, \pm 6579715)$, $(37090, \pm 7141335)$, $(39575, \pm 7871260)$, $(42584, \pm 8786173)$, $(48430, \pm 10656735)$, $(51644, \pm 11735203)$, $(89290, \pm 26680665)$, $(102940, \pm 33027165)$, $(123671, \pm 43490956)$, $(125074, \pm 44233143)$, $(136540, \pm 50453085)$, $(153260, \pm 59998685)$, $(163574, \pm 66156143)$, $(202996, \pm 91459869)$, $(232070, \pm 111796465)$, $(252586, \pm 126944409)$, $(354160, \pm 210765315)$, $(1593475, \pm 2011490010)$, $(2018755, \pm 2868305610)$, $(3952720, \pm 7858579965)$, $(787235, \pm 698484190)$, $(6353960, \pm 16016466685)$, $(6393934, \pm 16167848223)$, $(13396870, \pm 49034868465)$, $(21928510, \pm 102686578815)$, $(48439015, \pm 337126548960)$, $(67932950, \pm 559913206865)$, $(86518310, \pm 804751956815)$, $(157294520, \pm 1972742444285)$

118 points for $k = 924227785225$
$(-9466, \pm 275727)$, $(-9106, \pm 411297)$, $(-8700, \pm 515485)$, $(-8476, \pm 561507)$, $(-7540, \pm 703965)$, $(-4740, \pm 904285)$, $(-1600, \pm 959235)$, $(-586, \pm 961263)$, $(-234, \pm 961361)$, $(771, \pm 961606)$, $(830, \pm 961665)$, $(1230, \pm 962335)$, $(2190, \pm 966815)$, $(3356, \pm 980829)$, $(4350, \pm 1003265)$, $(7730, \pm 1177335)$, $(9495, \pm 1334260)$, $(10775, \pm 1474860)$, $(13184, \pm 1793277)$, $(15714, \pm 2191913)$, $(16035, \pm 2246590)$, $(19010, \pm 2791785)$, $(21800, \pm 3359235)$, $(22074, \pm 3417607)$, $(27656, \pm 4698621)$, $(31895, \pm 5776740)$, $(35715, \pm 6817690)$, $(36350, \pm 6996735)$, $(42410, \pm 8786535)$, $(46311, \pm 10012384)$, $(48420, \pm 10697885)$, $(58284, \pm 14103773)$, $(75450, \pm 20747015)$, $(87540, \pm 25918435)$, $(95430, \pm 29495665)$, $(116135, \pm 39588840)$, $(154640, \pm 60818685)$, $(289559, \pm 155816652)$, $(296280, \pm 161272835)$, $(372579, \pm 227421458)$, $(391320, \pm 244794365)$, $(461411, \pm 313425234)$, $(587175, \pm 449937640)$, $(740136, \pm 636748291)$, $(765330, \pm 669535415)$, $(830810, \pm 757273785)$, $(1725600, \pm 2266784515)$, $(1843920, \pm 2503877885)$, $(1978670, \pm 2783300385)$, $(2007704, \pm 2844785667)$, $(2064386, \pm 2966104041)$, $(10419734, \pm 33634497873)$, $(23134040, \pm 111269778435)$, $(29834264, \pm 162956989437)$, $(234788930, \pm 3597629410665)$, $(473207960, \pm 10293855372285)$, $(611677415, \pm 15128074966560)$, $(1622132880, \pm 65332554745085)$, $(2686813335, \pm 139269572546660)$

122 points for $k = 1272208985225$
$(-10834, \pm 23711)$, $(-10660, \pm 246685)$, $(-10210, \pm 455935)$, $(-9949, \pm 536126)$, $(-9245, \pm 694290)$, $(-8714, \pm 781359)$, $(-8480, \pm 813885)$, $(-6016, \pm 1026877)$, $(-5785, \pm 1038560)$, $(-5546, \pm 1049583)$, $(-2710, \pm 1119065)$, $(2050, \pm 1131735)$, $(4135, \pm 1158840)$, $(4696, \pm 1172931)$, $(6280, \pm 1232835)$, $(8476, \pm 1371549)$, $(8944, \pm 1409853)$, $(11614, \pm 1684863)$, $(12940, \pm 1854435)$, $(15550, \pm 2243265)$, $(16180, \pm 2346915)$, $(23011, \pm 3668334)$, $(25870, \pm 4311135)$, $(27220, \pm 4630365)$, $(27380, \pm 4668835)$, $(36515, \pm 7068190)$, $(40855, \pm 8334540)$, $(44900, \pm 9580765)$, $(45610, \pm 9805785)$, $(47939, \pm 10556662)$, $(54016, \pm 12604611)$, $(57020, \pm 13662365)$, $(79430, \pm 22414415)$, $(80464, \pm 22852413)$, $(98366, \pm 30871489)$, $(123415, \pm 43370940)$, $(161674, \pm 65016807)$, $(237095, \pm 115452760)$, $(281315, \pm 149211310)$, $(304151, \pm 167742724)$, $(356024, \pm 212434493)$, $(462130, \pm 314158665)$, $(543395, \pm 400567010)$, $(557446, \pm 416203569)$, $(751895, \pm 651983260)$, $(862990, \pm 801694815)$, $(928586, \pm 894815591)$, $(968840, \pm 953626685)$, $(1232810, \pm 1368813785)$, $(2436410, \pm 3802993465)$, $(7007930, \pm 18551739335)$, $(7740800, \pm 21536680765)$, $(8827171, \pm 26226015294)$, $(14467760, \pm 55030308035)$, $(19530286, \pm 86310355359)$, $(35313559, \pm 209851575348)$, $(53054066, \pm 386436384311)$, $(76346500, \pm 667088875485)$, $(169602340, \pm 2208756089565)$, $(745472515, \pm 20353891446810)$, $(3377879800, \pm 196320787319235)$

Largest computed examples

1. $\kappa = -(10^{15} + 25)$
   There are no points.

2. $\kappa = 10^{15} + 25$
   We calculated four points:

   $(134650, \pm 58662495)$, $(27850, \pm 31962495)$, $(-100000, \pm 5)$, $(9000000000000200000, \pm 270000000000000900000000000005)$.

   This list is not guaranteed to be complete, however.

3. $\kappa = \pm(3 * 10^{16} + 75)$
   There do not exist points.

4. $\kappa = -825399814482565487$ of almost 18 digits. According to Broadhurst (`http://physics.open.ac.uk/~dbroadhu/cert/`, mwrank12.txt (see description in "curves with many points")) the corresponding Mordell equation should have 158 integral points. It represents an elliptic curve of rank 12.

   The coresponding quadratic field E has discriminant

$$d_E = 2046445821031154103 \cdot 557 \cdot 1429567 \cdot 8565813 \; .$$

   Its class group Cl_E is isomorphic to $C_3 \times C_3 \times C_6 \times C_{36}$ of order 1944. $\Delta = -108\kappa$ factors into $d_E 66^2$. There are eight possibilities for the conductor f, and for each f we obtain from class field theory the number $N(F)$ of non-isomorphic fields F of dicriminant $d_E f^2$: $(f,\, N(F)) \in \{(1,40),\, (2,81),\, (3,81),\, (11,81),\, (6,162),\, (22,162),\, (33,162),\, (66,324)\}$.

   The calculation of generating polynomials for all 1093 non-isomorphic fields F was a formidable task. Those polynomials are listed below. The first entry of each row denotes the number of the field F with respect to the conductor f, the second the index I of the generating monic cubic polynomial with coefficients in columns 3 to 5. For each conductor the polynomials are ordered with respect to the index I.

$f = 1$

i	I	δ	p	q
1.	18	1	−38464710	91806797772
2.	25	1	−1976895	−819724650
3.	33	0	−17176932	27385940393
4.	39	0	−2026644	283469535
5.	49	1	−2739611	1106521626
6.	59	1	−2836161	860091996
7.	66	0	−3638013	1957466664

i	I	δ	p	q
8.	66	1	−12233782	−16373373940
9.	66	1	−20736706	36293737316
10.	67	1	−2985119	732739536
11.	69	0	−46006314	120093623325
12.	75	1	−9210241	−10561642030
13.	147	0	−5127792	1896481323
14.	311	1	−20515739	−34733598222
15.	430	1	−9819506	−358921356
16.	525	0	−19433382	29637340731
17.	553	1	−15992445	19338141162
18.	555	1	−16861405	−21839756872
19.	582	1	−15418042	−16923965236
20.	703	1	−14107117	−6434243884
21.	858	0	−25457337	43430642812
22.	918	1	−19891274	22954737036
23.	1173	0	−19194162	2178114093
24.	1214	1	−21064726	16353671996
25.	1279	1	−20903181	−10648247262
26.	1289	1	−30378781	53786672426
27.	1533	1	−27622469	36612263646
28.	1543	1	−24452427	−19020538734
29.	1581	0	−26488428	29306088171
30.	1782	1	−28775274	33501169116
31.	1842	0	−27336177	21321589852
32.	1992	0	−28424187	19865505862
33.	2069	1	−38495891	−72172626954
34.	2349	1	−34643061	44455469598
35.	2409	1	−52789341	−131909008374
36.	2709	1	−36373519	−39598561432
37.	2723	1	−36447577	−39421308766
38.	2800	1	−34907776	−18972149260
39.	3112	1	−36945760	11427236612
40.	3147	1	−39331335	38813695866

$$f = 2$$

i	I	δ	p	q
1.	3	1	−15289551	23005561383
2.	21	0	−3558084	2310054714
3.	23	1	−43040039	108660144759
4.	30	0	−3583872	2022603236
5.	33	0	−5006346	3909925818
6.	33	1	−5098721	4040035581
7.	33	1	−10244261	−12492207279
8.	33	1	−37905539	−89820270717
9.	33	1	−76127609	−255678104331
10.	33	0	−87343008	314183278674
11.	45	1	−3620691	943531659
12.	58	1	−7207149	6725301879
13.	61	1	−27007545	53909121981
14.	62	1	−4387269	−926934297
15.	87	1	−614660981	−5865656227179
16.	98	1	−6283669	−2766066121
17.	98	1	−146311429	681115510943
18.	111	0	−9804546	10113136390
19.	142	1	−7734441	−2725354149
20.	143	1	−20071327	−33710016385
21.	166	1	−8480325	−2611977873
22.	173	1	−9667137	6562090245
23.	294	0	−13736376	11042166700
24.	294	1	−27047405	−51674595273
25.	347	1	−28921119	56724202119
26.	359	1	−20572595	−29993259933
27.	381	0	−18353052	21811652342
28.	383	1	−20103607	−27556153225
29.	402	0	−50147028	134879269724
30.	406	1	−30005605	−59191903249

i	I	δ	p	q
31.	413	1	−15739767	−7787202225
32.	557	1	−24869589	36572773053
33.	699	0	−24825456	28023944410
34.	789	0	−39042942	83244935502
35.	891	1	−29803399	−38933039509
36.	891	1	−62265565	−182658746143
37.	891	1	−80913999	−275843255049
38.	922	1	−83303597	−288237546465
39.	936	1	−55041077	148464960579
40.	942	0	−29252988	31910638364
41.	969	0	−64278414	191045748806
42.	1034	1	−66843105	202471557987
43.	1063	1	−46693367	−107979872985
44.	1162	1	−31025541	−18197371977
45.	1174	1	−30525689	5936579931
46.	1174	1	−60060297	−167106688341
47.	1181	1	−30956055	−12899928849
48.	1183	1	−35145871	46771520603
49.	1294	1	−58167809	155159843211
50.	1326	0	−43181856	81229315780
51.	1455	0	−47183058	95620458682
52.	1621	1	−80192725	−261627500659
53.	1671	0	−44505414	67781340026
54.	1797	1	−53623589	114233340873
55.	1914	0	−73952244	220930877836
56.	1946	1	−64670565	169060215351
57.	1992	1	−68017557	185956752195
58.	1993	1	−50634459	84777645099
59.	1994	1	−63986809	163556503451
60.	2077	1	−45347659	27122857439
61.	2138	1	−48570981	−55849032177
62.	2145	1	−58015061	−122407037211
63.	2503	1	−82105869	250984314669
64.	2557	1	−52865851	−45474422257
65.	2834	1	−54777245	168511551

i	I	δ	p	q
66.	2854	1	−69446569	−157897567189
67.	2886	0	−73899684	185843363444
68.	3005	1	−84745765	−250608718375
69.	3007	1	−56985431	−1260356457
70.	3129	1	−65522255	−109524251733
71.	3491	1	−62963005	−5528602459
72.	3529	1	−67878119	92580222711
73.	3547	1	−64125247	−30373450525
74.	4125	1	−70355271	−2250970821
75.	4125	0	−72025728	61382490102
76.	4209	1	−71442117	17602126389
77.	4417	1	−77326957	−96722331235
78.	4622	1	−76696021	−45514842217
79.	4737	1	−81349661	108257624973
80.	4739	1	−77358807	22257285231
81.	5627	1	−86679839	−22109727741

$f = 3$

i	I	δ	p	q
1.	10	0	−52793037	147640765216
2.	13	0	−26132766	51408091407
3.	22	0	−51117741	140659594288
4.	28	0	−3790503	1649346134
5.	28	0	−5854383	4937425474
6.	41	0	−10487664	12626532691
7.	71	0	−21859794	38898924179
8.	104	0	−11512827	12340518842
9.	113	0	−8391966	672858843
10.	131	0	−35544366	80844288095
11.	139	0	−15312204	20001919529
12.	167	0	−102679098	400233503949
13.	176	0	−12041019	6879814486
14.	176	0	−14604819	15818075886

i	I	δ	p	q
15.	176	0	−67389939	212435136754
16.	176	0	−77240067	260878887102
17.	176	0	−86614452	309924915712
18.	211	0	−16861542	20162161617
19.	231	0	−51255966	139947786515
20.	247	0	−23325324	38261217165
21.	319	0	−16996824	5769119909
22.	319	0	−33194166	68734046325
23.	321	0	−18524154	15453305899
24.	321	0	−20183592	22698712357
25.	383	0	−23249538	29346031019
26.	435	0	−30778572	55034401439
27.	503	0	−24391638	20591428189
28.	579	0	−55699464	152688267281
29.	644	0	−43692207	97610171262
30.	678	0	−30625293	33461001704
31.	800	0	−101446587	387692050666
32.	860	0	−126694143	544271746642
33.	904	0	−109473099	434500347786
34.	1011	0	−39506844	46504993759
35.	1030	0	−84462957	286410430856
36.	1085	0	−38205852	15218063121
37.	1216	0	−62632539	162212641034
38.	1293	0	−53363766	105397452367
39.	1382	0	−45072333	23166450664
40.	1382	0	−45996621	37261919688
41.	1403	0	−60145524	137134711175
42.	1408	0	−49943763	70233557906
43.	1570	0	−107724537	410347849916
44.	1576	0	−58167099	110512398074
45.	1589	0	−52948062	69046073287
46.	1651	0	−52280946	50756904785
47.	1687	0	−109301244	417178034835

i	I	δ	p	q
48.	1700	0	−84263247	262530070114
49.	1745	0	−52678998	29764498747
50.	1776	0	−72973371	189876102230
51.	1822	0	−53472021	2286817032
52.	1895	0	−54886878	497473427
53.	1990	0	−76522797	198421384136
54.	2011	0	−66284586	124733405815
55.	2129	0	−75000042	177709756507
56.	2192	0	−77645763	191243755006
57.	2253	0	−73122438	152632557839
58.	2269	0	−74816766	164084004095
59.	2359	0	−115068222	433307383133
60.	2399	0	−97791012	315097882747
61.	2470	0	−84141873	215954766628
62.	2593	0	−103571586	344572602997
63.	2721	0	−71175234	53965038859
64.	2724	0	−70880871	46268459030
65.	2793	0	−80001066	150470931683
66.	2827	0	−78350688	129381983959
67.	2947	0	−87109476	196675970223
68.	3145	0	−91516182	214660743069
69.	3277	0	−117143676	406075249633
70.	3946	0	−92005629	95737009224
71.	3959	0	−90761256	62015891315
72.	4193	0	−118136826	352604063093
73.	4297	0	−96083496	73898258893
74.	4395	0	−96203208	11920889207
75.	4801	0	−136000152	464143151963
76.	4901	0	−111633714	205551282121
77.	5297	0	−115458246	191364438107
78.	5675	0	−118468422	163193908889
79.	6571	0	−130225824	180671562761
80.	6701	0	−149758344	437355012161
81.	8639	0	−153413172	160657779803

$f = 11$

i	I	δ	p	q
1.	6	0	−27894273	56675792844
2.	6	1	−41731026	103731791436
3.	6	0	−305376621	2054005591968
4.	30	1	−26491230	51679710828
5.	48	1	−12415080	8492465232
6.	48	1	−95747912	−360352406064
7.	78	0	−205956729	1137413707804
8.	83	1	−89367659	324173438136
9.	161	1	−364220809	2674874471828
10.	165	0	−54332388	145823857487
11.	187	1	−552193099	−4994277414304
12.	195	0	−320245728	2205044931177
13.	238	1	−52815022	−128978217436
14.	271	1	−39039909	−45616216362
15.	273	0	−173758464	877705682579
16.	324	1	−100173476	−373253204304
17.	349	1	−46833651	63607832418
18.	353	1	−285266945	1851314570796
19.	354	0	−62815401	158828377708
20.	384	1	−69861392	192305090580
21.	384	1	−241034064	−1435720591296
22.	422	1	−75790194	219437933196
23.	456	0	−93833091	321442276562
24.	488	1	−130984080	−557796891984
25.	597	0	−214228872	1193264241697
26.	663	1	−65784407	−43180035054
27.	671	1	−84402501	218567860806
28.	699	0	−96217308	295218528209
29.	737	1	−209330887	−1144234324774
30.	748	1	−550034580	−4960171737204

i	I	δ	p	q
31.	768	1	−72771992	−54800231664
32.	905	1	−79999531	27118183244
33.	908	1	−165242612	769897797744
34.	927	1	−113100379	−368171763874
35.	1056	0	−188679051	944897308102
36.	1338	1	−103490262	−4821336324
37.	1469	1	−128584771	342094372178
38.	1490	1	−130496890	354385861388
39.	1499	1	−207997027	−1061690770120
40.	1548	1	−125053804	−264537432964
41.	1738	1	−170711922	678181911636
42.	1779	0	−216925236	1105445147257
43.	1791	1	−259371889	−1513640634046
44.	1805	1	−227939961	−1206608286336
45.	1825	1	−158111125	−529317366250
46.	1849	1	−171550749	659041176198
47.	2001	0	−513633978	4439355469219
48.	2008	1	−141205984	−217601088124
49.	2031	0	−298177638	1883943404739
50.	2043	1	−272688995	−1619104722534
51.	2045	1	−374240445	2716782658500
52.	2058	0	−275212509	1643087597184
53.	2162	1	−309421482	1989906512196
54.	2308	1	−216474892	1007066624144
55.	2511	1	−158096319	84577799304
56.	2535	0	−206527662	845993618459
57.	2831	1	−170965571	−72861699594
58.	2913	0	−196157412	583057291267
59.	3156	0	−186040503	201150772966
60.	3578	1	−305270610	−1743792363348
61.	3687	1	−278097607	1392599645864
62.	4039	1	−422431709	3109782201414
63.	4231	1	−235132861	532986028496
64.	4674	0	−257665221	728551267048
65.	4761	1	−359481261	−2191771556064

i	I	δ	p	q
66.	4803	1	−268501119	−867291746004
67.	5019	0	−452599488	3380099482639
68.	5271	0	−349516464	1943582435685
69.	5295	0	−524766528	4340231129773
70.	5892	0	−389533551	2360655490050
71.	7479	1	−378750379	−1708736930122
72.	7534	1	−328595054	224973131964
73.	8805	1	−367236745	476419546568
74.	8850	0	−370105593	572304852108
75.	9675	1	−438325455	−1972870190172
76.	9939	1	−395884991	363593081916
77.	10851	0	−424865628	750634688969
78.	11065	1	−440526181	−1198731302056
79.	12146	1	−470426766	1375857219108
80.	12853	1	−549178769	3063681613056
81.	14010	0	−507090177	1147378751596

$f = 6$

i	I	δ	p	q
1.	3	0	−1480224	484675234
2.	5	0	−1681902	150678346
3.	5	0	−9371178	11010855002
4.	7	0	−57961284	169841980614
5.	11	0	−2835396	274681618
6.	11	0	−2858238	397147286
7.	11	0	−2913936	603263058
8.	11	0	−3149226	1151298502
9.	11	0	−3535524	1801563410
10.	11	0	−4228998	2811284106
11.	11	0	−5853978	5139887866
12.	11	0	−6901458	6737748774

i	I	δ	p	q
13.	11	0	−12502176	16917487698
14.	11	0	−26470548	52387954706
15.	11	0	−32825556	72365136878
16.	11	0	−33402438	74282322314
17.	11	0	−54011274	152772093510
18.	11	0	−82542186	288637967558
19.	11	0	−101470956	393419706222
20.	11	0	−118736766	497992780438
21.	11	0	−294093006	1941220942422
22.	25	0	−22469478	40787107602
23.	29	0	−11565906	14361872682
24.	39	0	−11233596	12981301882
25.	39	0	−11254476	13026413702
26.	83	0	−37922334	88833945126
27.	119	0	−18186594	22466703810
28.	137	0	−15243084	3546064294
29.	137	0	−47254932	122966263258
30.	169	0	−33379056	68776919642
31.	182	0	−27715236	47435322300
32.	183	0	−26173074	41742291454
33.	193	0	−44689086	110479555390
34.	203	0	−42711966	102074827370
35.	205	0	−81549432	281422003394
36.	214	0	−67489224	210454496220
37.	281	0	−41933466	93645045182
38.	317	0	−26570382	6088483002
39.	326	0	−36458448	65418667836
40.	353	0	−39618204	76239694794
41.	375	0	−44469858	95871712318
42.	403	0	−39110874	66570873366
43.	421	0	−100059828	378916905714
44.	434	0	−45629748	94526559916
45.	447	0	−92596512	334914517942
46.	466	0	−49253028	108515347476
47.	491	0	−117532368	483685397906

i	I	δ	p	q
48.	547	0	−120010416	497898049890
49.	582	0	−48738036	88932696100
50.	582	0	−190523712	1007636638012
51.	609	0	−274933938	1751764480846
52.	615	0	−41574198	18036571822
53.	619	0	−111913488	444072775826
54.	634	0	−60586236	148254588572
55.	634	0	−71544276	208050221732
56.	718	0	−45624672	1867103372
57.	803	0	−276476766	1764462259770
58.	843	0	−224223564	1284795577874
59.	861	0	−82247898	249398428106
60.	893	0	−54191034	42629895286
61.	906	0	−63725376	126257939228
62.	931	0	−54247644	168477790
63.	1011	0	−64647336	110168100542
64.	1067	0	−61758324	61901710546
65.	1154	0	−91962684	280862491340
66.	1154	0	−98934984	327303907220
67.	1285	0	−90430062	253969289034
68.	1290	0	−114062808	417665463068
69.	1375	0	−70716132	28335708106
70.	1375	0	−151455978	680524851602
71.	1501	0	−81641076	138350829858
72.	1518	0	−133964436	541572206060
73.	1563	0	−141632634	595187664806
74.	1615	0	−156919122	708000020014
75.	1667	0	−85800222	133231902842
76.	1669	0	−116378946	396876387002
77.	1779	0	−89367888	139228828994
78.	1823	0	−99225576	232492321930
79.	1914	0	−233543568	1336848048196
80.	1929	0	−108091434	292517141150
81.	2117	0	−113812056	310027592050
82.	2298	0	−177695292	828941253532

i	I	δ	p	q
83.	2338	0	−108256272	196987410132
84.	2342	0	−105643824	158150873604
85.	2761	0	−111999024	11313061990
86.	2785	0	−172377018	739713660242
87.	2815	0	−129253812	322007651466
88.	2874	0	−139382364	419265318940
89.	2923	0	−118290894	109943623446
90.	2939	0	−116836056	24322577422
91.	2945	0	−136660212	376114472566
92.	3013	0	−161779824	616104652174
93.	3171	0	−164995248	625360501474
94.	3182	0	−128348964	192259273324
95.	3261	0	−170932728	670623623134
96.	3279	0	−151765734	473805736130
97.	3301	0	−129269106	150659931962
98.	3530	0	−135386268	166223669892
99.	3562	0	−158404164	492583433604
100.	3731	0	−151102818	362324805386
101.	3773	0	−146896854	284914425126
102.	3773	0	−175133832	638259058722
103.	3783	0	−158949372	452158985158
104.	3801	0	−159477618	454614360226
105.	3837	0	−139552554	30232654666
106.	3874	0	−152733996	344001794828
107.	4217	0	−153103392	215527357562
108.	4253	0	−168428466	462929790550
109.	4261	0	−168602328	463292367566
110.	4295	0	−176148072	553503623686
111.	4390	0	−165631428	383812594148
112.	4430	0	−154032972	77070319764
113.	4521	0	−192241758	703443319954
114.	4547	0	−159440604	190603664994
115.	4610	0	−254862348	1368444513972
116.	4651	0	−200634786	778628286682
117.	4666	0	−172033308	400280798236

i	I	δ	p	q
118.	4901	0	−166199868	157230034574
119.	4945	0	−190616382	599037473506
120.	5029	0	−180902538	432408769306
121.	5155	0	−169870038	32961682662
122.	5213	0	−178869354	326060037366
123.	5287	0	−221892282	925111294142
124.	5290	0	−174920388	171247695612
125.	5331	0	−178128786	248755110838
126.	5366	0	−174456648	30635219684
127.	5366	0	−212807988	801314844604
128.	5746	0	−183684348	131381727124
129.	6122	0	−254150964	1187180578564
130.	6269	0	−199706436	328064728162
131.	6437	0	−282028164	1480790083854
132.	6458	0	−278843004	1440145294564
133.	6689	0	−207493974	320340936350
134.	6701	0	−309905658	1784438723606
135.	6773	0	−212330754	408030787174
136.	6783	0	−228671022	718367263742
137.	7031	0	−213000288	287698133274
138.	7225	0	−228873318	593140099758
139.	7297	0	−239369274	760935423634
140.	7373	0	−280687704	1338982814526
141.	7619	0	−266855706	1109675847542
142.	7746	0	−264347688	1048571140844
143.	7823	0	−239266926	599507371550
144.	8113	0	−230307264	117347587754
145.	8291	0	−234638028	195251203874
146.	8359	0	−237064788	259280296454
147.	8363	0	−240518382	391058337558
148.	8454	0	−239282616	282064189372
149.	8699	0	−260365356	741650745038
150.	9286	0	−313502748	1487253551844
151.	9337	0	−252608364	96182580154
152.	9339	0	−252399324	48045215950

i	I	δ	p	q
153.	9489	0	−268508424	641157087850
154.	9777	0	−263747424	331384473686
155.	10289	0	−271703286	288009036542
156.	10330	0	−272250648	279065287172
157.	10690	0	−288279252	656595030004
158.	11182	0	−344580444	1627751967916
159.	11623	0	−302216076	634924070150
160.	11806	0	−333393444	1298760123980
161.	12271	0	−306445284	393372918230
162.	13889	0	−576766674	4812606016270

$f = 22$

i	I	δ	p	q
1.	3	1	−2911971	−598061169
2.	3	1	−2935181	−667827459
3.	3	1	−2977289	778907589
4.	3	0	−4117938	2653963686
5.	3	1	−4225899	2805494679
6.	3	0	−6609174	6282376650
7.	3	1	−7957451	−8449332597
8.	3	0	−8970324	10180058130
9.	3	1	−22364679	40661132499
10.	3	1	−34130939	76715713083
11.	3	1	−48346107	−129390278085
12.	3	0	−56170158	162023971626
13.	3	0	−105376524	416351161710
14.	3	1	−173971189	−883266395167
15.	3	0	−303643626	2036545859898
16.	3	1	−619070151	−5928884725689
17.	3	1	−1191225031	−15825218823097

i	I	δ	p	q
18.	57	1	−25262409	−34600280211
19.	81	1	−36495917	69231863589
20.	81	1	−64932897	195304865589
21.	94	1	−119491005	499474729191
22.	109	1	−47126647	105565455191
23.	151	1	−182076935	941156535867
24.	174	0	−359363664	2619981911404
25.	178	1	−252051257	1536353383035
26.	207	1	−63566191	149420607191
27.	219	0	−59738712	118277363358
28.	231	0	−53844516	59597738090
29.	231	0	−98563212	349683502058
30.	258	0	−79404336	223050386548
31.	283	1	−143237519	−637227408501
32.	375	0	−82225572	175421279286
33.	375	0	−104188722	340540577586
34.	375	1	−123753285	−478781979975
35.	375	0	−404701638	3125408443938
36.	375	1	−689514481	−6965411029975
37.	375	1	−877873135	−10009137490717
38.	399	1	−186515215	−950248430089
39.	414	1	−376033137	−2795545435509
40.	445	1	−318996641	2176210102845
41.	445	1	−1014935841	−12442737416991
42.	478	1	−82709249	−2032275621
43.	518	1	−114085209	−348674501661
44.	525	0	−93119598	136061930422
45.	557	1	−171564169	796381801829
46.	591	0	−133573506	474273440190
47.	633	0	−866611548	9811909500074
48.	647	1	−133174279	443065532699
49.	717	0	−217705158	1157600693546
50.	725	1	−109204795	−10854157525
51.	742	1	−724061917	7485423888215
52.	781	1	−146198895	−489112419633

i	I	δ	p	q
53.	822	0	−432266868	3423187682596
54.	858	1	−400263321	3037984626411
55.	899	1	−126367765	−49746784639
56.	967	1	−134454261	130589155101
57.	1029	0	−137942376	20654854110
58.	1029	1	−161099209	480547632761
59.	1029	1	−206169157	953790283169
60.	1029	1	−368025225	−2645156583819
61.	1138	1	−162814769	405319329963
62.	1267	1	−363671117	−2556823945095
63.	1285	1	−315070301	−2007057538851
64.	1367	1	−235088201	−1113314990559
65.	1382	1	−356305737	−2449759918245
66.	1403	1	−337219219	2226771078959
67.	1459	1	−174409935	71233323951
68.	1538	1	−374413277	2628208770735
69.	1574	1	−184998645	−170794820913
70.	1639	1	−197486197	394386957149
71.	1669	1	−202049767	447294798839
72.	1703	1	−662858929	−6487436412151
73.	1719	1	−226404977	796981000341
74.	1739	1	−206965865	−451737348999
75.	1758	1	−231725281	−842473143877
76.	1905	0	−352280412	2268376279334
77.	2034	1	−472588277	−3757684123041
78.	2038	1	−430496777	−3208884457605
79.	2047	1	−584290109	5292677996769
80.	2187	1	−241086501	−566944835679
81.	2194	1	−648727449	6219176070171
82.	2202	1	−429217741	−3152267911729
83.	2202	0	−817091748	8890408066364
84.	2257	1	−251586331	−700395660769
85.	2321	1	−569654609	5040638438361
86.	2337	0	−714826926	7218664756622
87.	2359	1	−624517439	5834500306611

i	I	δ	p	q
88.	2393	1	−333091161	1836816430161
89.	2461	1	−273507835	−899722352773
90.	2678	1	−318675529	−1471023520837
91.	2685	0	−275153472	664466866514
92.	2685	1	−624511895	5782493940375
93.	3171	0	−614923452	5546072972242
94.	3195	1	−312705235	−886264436125
95.	3417	0	−793900998	8357450738626
96.	3567	0	−397930434	2160447237630
97.	3606	0	−647995392	5961523314428
98.	3795	0	−502070382	3669633926794
99.	4011	0	−348226164	594918885374
100.	4042	1	−357560969	882516010803
101.	4071	1	−372331065	−1251982455759
102.	4197	0	−478391958	3123778825574
103.	4379	1	−509884459	3550065345671
104.	4553	1	−676174979	6180084808599
105.	4585	1	−391580075	1087693774275
106.	4689	1	−1001015187	11854374088599
107.	4806	1	−446327977	2167561959539
108.	4834	1	−389379789	−417124069089
109.	5307	1	−433543491	−1319432712237
110.	5315	1	−525402841	3335063540345
111.	5430	1	−501193645	2798956605575
112.	5619	0	−430049994	447758400474
113.	5638	1	−518486041	−2998255876189
114.	5691	1	−680262777	5895142466409
115.	5743	1	−730958037	6764375565165
116.	5879	1	−467000145	−1552390169739
117.	6182	1	−469511137	1145925154955
118.	6322	1	−519452149	2470277242079
119.	6414	1	−531885845	2683173671847
120.	6691	1	−677028139	5435857634711
121.	6926	1	−529778905	2104800629171
122.	7013	1	−502719081	883055623701

i	I	δ	p	q
123.	7138	1	−944229909	−10297255881057
124.	7178	1	−753132189	6661966987839
125.	7191	1	−642759355	−4513632462253
126.	7413	0	−633102228	4175510986306
127.	7518	0	−523837116	749132709292
128.	7518	0	−629997588	4038491342996
129.	8394	0	−641916132	3652183080612
130.	8413	1	−617659027	2990563057535
131.	8494	1	−1008561077	−11203846278561
132.	8830	1	−936062585	−9639151758117
133.	8951	1	−703660679	−4714520116869
134.	9007	1	−695026911	4469536661511
135.	9141	0	−683312652	4076013939778
136.	9254	1	−964100397	10066626211431
137.	9643	1	−839631899	7319603510283
138.	9737	1	−725656477	4671942690545
139.	10098	1	−645151349	1540731651039
140.	10142	1	−750621241	4991920440923
141.	10237	1	−915467571	−8673291288609
142.	10623	1	−654621447	−402057798705
143.	10702	1	−756641497	−4707630197965
144.	11379	0	−1077532434	11740874083334
145.	11509	1	−690352989	−389585212359
146.	11579	1	−909267817	7885562531669
147.	11843	1	−751136987	3366100322715
148.	11890	1	−720466985	−1882288346325
149.	12106	1	−713677917	−298872560769
150.	13626	1	−1042248225	9980553840651
151.	13666	1	−1054905045	10266246013791
152.	15189	0	−1053759564	9418901062274
153.	15393	0	−1158350226	11972338159518
154.	17133	0	−900429156	686432743282
155.	18102	0	−953244864	2848873073860
156.	19410	0	−1017356208	4217672355732
157.	19653	0	−1050617238	5487486494886

i	I	δ	p	q
158.	19914	0	−1112063436	7633185834340
159.	19914	0	−1181427984	9940744669636
160.	21882	0	−1091069496	4094913997988
161.	24027	0	−1186271988	5961220504866
162.	27783	1	−1270781519	4566781164999

$f = 33$

i	I	δ	p	q
1.	2	0	−3453153	1672900284
2.	2	0	−4280001	2883336852
3.	2	0	−6552513	6194969836
4.	2	0	−10571649	13104701940
5.	2	0	−13688961	19409262708
6.	2	0	−338429949	2396355087720
7.	2	0	−545089149	4898345210440
8.	2	0	−653980833	6437179602524
9.	2	0	−1856987649	30800774391860
10.	11	0	−20020176	32998519235
11.	16	0	−12093147	7120775882
12.	16	0	−37054347	85591830918
13.	16	0	−70575219	227742252914
14.	16	0	−141550827	648048488538
15.	16	0	−393953571	3009612695650
16.	16	0	−1503456867	22438017480098
17.	41	0	−25096896	30892357265
18.	66	0	−61143357	173979916792
19.	93	0	−161640336	786467610907
20.	101	0	−98323236	363868408915
21.	128	0	−116845443	472031319166
22.	128	0	−117993051	479422357238

i	I	δ	p	q
23.	128	0	−479395323	4038395187926
24.	128	0	−1550218659	23492662130850
25.	131	0	−46918674	33716360821
26.	185	0	−181370772	925008384289
27.	193	0	−72408996	159682097047
28.	250	0	−168082233	807408538068
29.	257	0	−107920296	362897074903
30.	304	0	−87572067	152358018978
31.	339	0	−965904654	11550350501701
32.	346	0	−602831097	5688260074948
33.	371	0	−725238162	7509864528133
34.	443	0	−157157286	642696200157
35.	509	0	−232991154	1288377666059
36.	527	0	−202130874	997113318025
37.	572	0	−1739828871	27927543641558
38.	677	0	−389597868	2895261512429
39.	686	0	−583268829	5385957433824
40.	841	0	−868967154	9829810441759
41.	857	0	−303836478	1883936276079
42.	871	0	−1480861734	21919824258321
43.	1024	0	−430876611	3314438908990
44.	1181	0	−324062244	1972439389019
45.	1284	0	−261182559	1130815045294
46.	1570	0	−516897117	4292506250824
47.	1645	0	−310179012	1479046708491
48.	1649	0	−354586122	2088156778103
49.	1771	0	−686555694	6734537774701
50.	2000	0	−1690151067	26682822073226
51.	2010	0	−532654617	4365115230476
52.	2011	0	−352897974	1781270252861
53.	2182	0	−358315353	1698701516996
54.	2221	0	−542625666	4427005482075
55.	2427	0	−2513164566	48442895012963

i	I	δ	p	q
56.	2491	0	−573741762	4781024194947
57.	2581	0	−476369322	3242926516467
58.	2867	0	−430890966	2251145143643
59.	2882	0	−466232481	2856317121988
60.	2923	0	−444708924	2444835966045
61.	2966	0	−749070729	7416656864716
62.	3071	0	−1225650492	16278370345303
63.	3112	0	−543907683	3980513601614
64.	3121	0	−726622182	6985406468887
65.	3139	0	−649903914	5703877252469
66.	3227	0	−809163036	8360195481517
67.	3354	0	−604378137	4839467171308
68.	3486	0	−410587617	473352578732
69.	3524	0	−415805247	632546188978
70.	3667	0	−461104776	1850739375947
71.	3954	0	−596497341	4305629266400
72.	3957	0	−490829196	2143392187453
73.	3976	0	−722700627	6547683984798
74.	4119	0	−1116317532	13859557758673
75.	4170	0	−460812693	379364843792
76.	4223	0	−507629826	2158546421977
77.	4244	0	−681938631	5667040424650
78 :	4561	0	−489057822	398210702263
79.	4597	0	−723708018	6221900693459
80.	4677	0	−895054908	9390125847949
81.	4885	0	−587657202	3220061723571
82.	5015	0	−522842562	644443455359
83.	5029	0	−575685924	2718493955859
84.	5073	0	−671872674	4867259005015
85.	5416	0	−667705632	4459797791592
86.	5665	0	−945635328	9939160663223
87.	5829	0	−1155791802	14166561788093
88.	5987	0	−705602304	4739081367845
89.	6024	0	−739760736	5479268981560
90.	6028	0	−626644359	2542211133770
91.	6081	0	−632519382	2639716121977

i	I	δ	p	q
92.	6275	0	−604129878	405839356373
93.	6312	0	−797785923	6506825138066
94.	6797	0	−933392988	9074150664469
95.	6843	0	−861157308	7480691363981
96.	6918	0	−784440921	5657657906428
97.	6919	0	−702209706	3432718660305
98.	7079	0	−887947434	7896599785679
99.	7095	0	−662853972	1263764128561
100.	7162	0	−679018329	2010672982780
101.	7431	0	−715916784	2963472913951
102.	7481	0	−685321494	1221035143119
103.	7647	0	−788245788	4928562957769
104.	7837	0	−827923338	5777548916821
105.	7873	0	−822728346	5598261024807
106.	7940	0	−768558087	3901165691434
107.	8089	0	−802760442	4757337622257
108.	8125	0	−749813262	2822063517141
109.	8175	0	−720861438	576825274487
110.	8192	0	−837117996	5613880742448
111.	8192	0	−890300268	7010987897936
112.	8556	0	−754294431	1776586949390
113.	9011	0	−1067088726	10629655646555
114.	9021	0	−835507002	4385974096757
115.	9203	0	−939657906	7280643958907
116.	9384	0	−795916704	1418464149784
117.	9812	0	−895368903	5184179598774
118.	9826	0	−821947137	1604631469188
119.	9987	0	−846288498	2733012325109
120.	10319	0	−853106364	2023015943641
121.	11310	0	−906267153	2165640141148
122.	11469	0	−933248514	3441756396859
123.	11562	0	−963518853	4709489710376
124.	11623	0	−1203788916	12121299507713
125.	11723	0	−967631148	4559493757181
126.	11930	0	−935905113	1993344668788

i	I	δ	p	q
127.	12444	0	−1216100919	11774027260006
128.	12479	0	−980045352	3304460755607
129.	12975	0	−1230269898	11700809570153
130.	13272	0	−1146111384	8811698485480
131.	13412	0	−1035219471	3985555692162
132.	13496	0	−1172593224	9408359620616
133.	13496	0	−1205979291	10464504736730
134.	14168	0	−1085396136	4873501462632
135.	14303	0	−1045651596	721982745033
136.	14559	0	−1621606404	21372321467959
137.	14807	0	−1090072764	3305742683375
138.	14964	0	−1118042607	4714183177022
139.	15005	0	−1521745038	18336384690613
140.	15115	0	−1317273012	12249697003709
141.	15433	0	−1123977228	3710937669681
142.	15465	0	−1126025472	3755941454639
143.	17048	0	−1339973448	10795793717544
144.	17237	0	−1278989118	8044605931219
145.	17664	0	−1202621772	378040531088
146.	17940	0	−1239949623	4096186270022
147.	18835	0	−1255129458	334761358107
148.	18911	0	−1270501584	2939299071159
149.	19861	0	−1330408134	4824223128659
150.	20029	0	−1307465874	58010541259
151.	20487	0	−1327569078	443503668751
152.	21207	0	−1363413216	2064049263727
153.	22475	0	−1469717898	7307155675347
154.	22480	0	−1431381732	4166139541344
155.	22592	0	−1435041036	4065801478288
156.	22882	0	−1469332161	6147127315868
157.	23215	0	−1446609978	1910471443177
158.	23393	0	−1497839706	6793913001337
159.	23431	0	−1479072474	5119461368879
160.	24932	0	−1514818239	1375778513010
161.	25025	0	−1614390228	10316406552727
162.	31504	0	−1972819332	17841153098912

$f = 66$

i	I	δ	p	q
1.	1	0	−2814174	13705450
2.	1	0	−2832126	252545898
3.	1	0	−3201858	1249551974
4.	1	0	−3485988	1724607214
5.	1	0	−4134174	2677007850
6.	1	0	−4564824	3284853330
7.	1	0	−4705074	3482736730
8.	1	0	−5123448	4077095506
9.	1	0	−11179608	14272388526
10.	1	0	−13502148	19009797906
11.	1	0	−16125516	24857724562
12.	1	0	−19348296	32707176018
13.	1	0	−32190576	70274232402
14.	1	0	−42370218	106139131846
15.	1	0	−43541586	110572174042
16.	1	0	−60801774	182473986090
17.	1	0	−82807098	290028674694
18.	1	0	−112722786	460640698342
19.	1	0	−267799356	1686794413678
20.	1	0	−435492486	3497993950858
21.	1	0	−581152176	5392409825198
22.	2	0	−4771404	1698976444
23.	19	0	−110711304	447038400490
24.	29	0	−1258551558	17185120805534
25.	33	0	−29805204	18087146066
26.	33	0	−48089514	113491813754
27.	33	0	−3164304396	68511792317650
28.	35	0	−151018362	711483277934
29.	42	0	−48080604	103163249684
30.	51	0	−136012998	603472905794
31.	74	0	−88081488	288374722636
32.	107	0	−79376154	190502357826
33.	118	0	−83658564	201916158356
34.	125	0	−79526502	151411626146
35.	125	0	−134264988	554067929162

i	I	δ	p	q
36.	125	0	−147929892	654213884874
37.	125	0	−458821638	3775980123938
38.	125	0	−634221918	6143464892158
39.	125	0	−749838672	7899879061514
40.	125	0	−1816259082	29792173696494
41.	134	0	−472345764	3943774196340
42.	170	0	−89253252	99587811796
43.	174	0	−156306216	682489061188
44.	177	0	−270747642	1684296570022
45.	206	0	−407131296	3139678945788
46.	206	0	−569934816	5223646412292
47.	215	0	−113856072	256985009886
48.	226	0	−116226924	252925149180
49.	246	0	−222619188	1197788368876
50.	262	0	−119953284	170487989364
51.	286	0	−190174248	865383621476
52.	290	0	−402263928	3060346437148
53.	330	0	−139371672	203777949364
54.	343	0	−155000142	404050554182
55.	343	0	−584942424	5409465195486
56.	343	0	−5581597626	160502805235190
57.	415	0	−158016672	126133197586
58.	442	0	−252359844	1317563322636
59.	443	0	−2995286184	63091413137434
60.	461	0	−2321784036	43052528027498
61.	469	0	−183702354	438400428510
62.	475	0	−4531570758	117411200428082
63.	506	0	−183814488	273445803796
64.	506	0	−2869454016	59155423600108
65.	525	0	−206256468	624438693142
66.	529	0	−1448007858	21186439900934
67.	537	0	−189211836	226883236030
68.	550	0	−652440228	6336118244148
69.	574	0	−3424261896	77118550704932
70.	638	0	−215224152	364781529948

i	I	δ	p	q
71.	638	0	−1048410792	13014552236708
72.	678	0	−651549756	6281655590540
73.	687	0	−249013974	853979267306
74.	723	0	−332667192	1930879747478
75.	723	0	−2046296604	35604555058006
76.	727	0	−3844227816	91731133977890
77.	849	0	−4508216724	116497664488370
78.	881	0	−6062683176	181688971999282
79.	885	0	−667347582	6437739645106
80.	903	0	−552489762	4721466577718
81.	926	0	−592145268	5284749894204
82.	979	0	−446993118	3172821578014
83.	1014	0	−1262058996	17158453071892
84.	1045	0	−1348898562	18973748548766
85.	1091	0	−895849614	10128322164290
86.	1123	0	−1432596924	20770570478966
87.	1245	0	−343567488	943648739638
88.	1365	0	−691352112	6542403659066
89.	1405	0	−721227012	7004409050166
90.	1689	0	−883687134	9634034846810
91.	1774	0	−1932731988	32545112374236
92.	2035	0	−2435771382	46122377991294
93.	2050	0	−1322973828	18143019420548
94.	2073	0	−2250455394	40918660604426
95.	2107	0	−721675482	6405126738978
96.	2157	0	−1801299324	29163525078314
97.	2177	0	−722071152	6334603564718
98.	2197	0	−1321197504	18047936159814
99.	2197	0	−2068882794	35999629886594
100.	2234	0	−960856776	10721276623508
101.	2294	0	−740332296	6537556131788
102.	2330	0	−883977468	9187486897908
103.	2357	0	−874330776	8982108380410
104.	2379	0	−1252068906	16495608655042

i	I	δ	p	q
105.	2397	0	−991149456	11192836378570
106.	2449	0	−1233090276	16061302920498
107.	2519	0	−1461961446	21023036743962
108.	2591	0	−2330966418	43059773244774
109.	2725	0	−699055632	5108129970394
110.	2747	0	−1381343634	19119844710274
111.	3009	0	−916106664	9165688234450
112.	3095	0	−597677652	61309429346
113.	3095	0	−984038352	10466165059746
114.	3229	0	−895106916	8474911176182
115.	3233	0	−883592622	8227460165782
116.	3387	0	−635610162	409007842562
117.	3401	0	−1463722128	20649540005534
118.	3446	0	−2026994244	34563266285460
119.	3459	0	−1069555278	11906261783746
120.	3610	0	−838974312	6667830618284
121.	3626	0	−2147337324	37729024453780
122.	3633	0	−1078319352	11923813373362
123.	3674	0	−2290049388	41649234027764
124.	3709	0	−812041956	5823219805238
125.	3745	0	−1619921952	24154915795054
126.	3782	0	−683481744	277887255404
127.	3835	0	−958968648	9060499365478
128.	3849	0	−1631220096	24374606344418
129.	3907	0	−2120738334	36914118014334
130.	4031	0	−736394604	2347854889710
131.	4039	0	−1135460898	12767769228874
132.	4089	0	−816196656	5034910609982
133.	4170	0	−729796188	415584585812
134.	4249	0	−2246648316	40253731043262
135.	4255	0	−948109338	8153902388038
136.	4287	0	−795853344	3741717822766
137.	4354	0	−1232142516	14647104771332
138.	4647	0	−797167272	1937060348542
139.	4843	0	−3029220744	63565614051654
140.	4857	0	−2486869572	46911047995262

i	I	δ	p	q
141.	4859	0	−1059807606	9919697040462
142.	4895	0	−1370152212	17376952229934
143.	4913	0	−1061350686	9870643207370
144.	4913	0	−1878671652	30043603609998
145.	4947	0	−1378718814	17534571639266
146.	5030	0	−952392012	6666855803916
147.	5070	0	−2318217528	41962159398148
148.	5118	0	−836846076	583779393340
149.	5253	0	−965685864	6504643819366
150.	5331	0	−873231546	2194796911538
151.	5403	0	−1482972942	19666821177938
152.	5518	0	−987626112	6495248111748
153.	5547	0	−1878811374	29680671959246
154.	5578	0	−1012368456	7140537368140
155.	5586	0	−2265531708	40245086941796
156.	5758	0	−1297843272	14642443144868
157.	5798	0	−922539816	2308686296620
158.	5815	0	−1253910372	13432604797986
159.	5893	0	−1115880084	9549497445466
160.	5973	0	−1909345152	30222930552518
161.	6127	0	−975020904	3657713777806
162.	6131	0	−964691046	2983037187262
163.	6145	0	−1078640442	7826130581126
164.	6189	0	−1184716434	10948928348270
165.	6335	0	−1528715628	19919044568302
166.	6391	0	−1582363464	21263003170530
167.	6469	0	−1095473016	7522996886042
168.	6753	0	−1220829654	10908751371914
169.	6753	0	−1527364344	19424307551506
170.	6859	0	−1037171718	3156577289426
171.	6859	0	−1080389376	5612421892858
172.	6859	0	−1298652036	13005514769082
173.	6859	0	−1391624916	15618572306502
174.	6859	0	−2279488266	39992440652098
175.	6861	0	−1035154956	2985137168618

i	I	δ	p	q
176.	6886	0	−1153384584	8411785779540
177.	6973	0	−1689747642	23542084848722
178.	6999	0	−1289484768	12486590489054
179.	7001	0	−1129176378	7174319142166
180.	7197	0	−2023749156	32509951376650
181.	7509	0	−2110774908	34742861456006
182.	7603	0	−2252285244	38753013654806
183.	7651	0	−1097874888	1666191250614
184.	7706	0	−1150022016	5410244813908
185.	7869	0	−1462617354	16096657941650
186.	8079	0	−1258501926	8933086588822
187.	8178	0	−1679380824	21928923292804
188.	8245	0	−1149885528	897071524602
189.	8507	0	−1203676254	4407541192274
190.	8533	0	−1702940382	22163907740942
191.	8542	0	−1518149952	16657332899812
192.	8553	0	−1202529306	4012182531830
193.	8751	0	−1207746606	2855420725322
194.	9137	0	−1237646124	2285613377806
195.	9199	0	−1277816166	5451152382038
196.	9242	0	−1257712632	3568795342508
197.	9254	0	−2105457948	33166066208396
198.	9466	0	−1272698856	3092346317588
199.	9605	0	−2058443838	31425445964462
200.	9651	0	−1458692796	12341109211882
201.	9717	0	−2460775746	43541035916510
202.	9813	0	−1426280592	10579077228742
203.	10141	0	−1571520456	15344337612362
204.	10158	0	−1321472064	1096283518396
205.	10275	0	−2401301892	41264588288374
206.	10520	0	−1419392832	7632667060656
207.	10569	0	−1953221754	27113699683510
208.	10718	0	−2152593564	33142324456956
209.	11042	0	−1476189396	8603217270340
210.	11045	0	−1629421398	15431797448078

i	I	δ	p	q
211.	11225	0	−4063963662	97609645528934
212.	11373	0	−1770170754	19867259470894
213.	11409	0	−1710565428	17656542075854
214.	11674	0	−1616621028	13265501143884
215.	11717	0	−1689000984	16141412045594
216.	11962	0	−1476646512	2141077585812
217.	11991	0	−2533271268	43974458719774
218.	12104	0	−1521133944	6142174682160
219.	12167	0	−1521773154	5773694548534
220.	12167	0	−1578183882	9673399060842
221.	12167	0	−1771717992	18307246471998
222.	12179	0	−1502479836	3572700363562
223.	12194	0	−1564414368	8734603919844
224.	12269	0	−1719981054	16026069774110
225.	12327	0	−1842330864	20608418726786
226.	12329	0	−1529864226	5348426308522
227.	12392	0	−1599604512	9968243208528
228.	12408	0	−1573166496	8275521956240
229.	12449	0	−3379911678	72170216445866
230.	12538	0	−1518792792	114238473492
231.	12678	0	−1570009584	6531184601876
232.	12754	0	−2340464544	36909154601660
233.	12858	0	−1657656264	11356046370196
234.	13040	0	−2070482832	27450801434144
235.	13040	0	−2249718768	33547847720992
236.	13222	0	−2298665556	34959878301260
237.	13573	0	−1785595482	15335629504162
238.	14167	0	−2610176514	44406110378454
239.	14174	0	−2682495024	46865253512220
240.	14181	0	−1911357228	19249031282074
241.	14221	0	−1876707096	17650134915798
242.	14311	0	−1919306598	19268036369274
243.	14479	0	−2066744196	24813169244402
244.	14574	0	−2172351324	28591710023140
245.	14631	0	−2576310528	42738319912574

i	I	δ	p	q
246.	15101	0	−2100067596	24884435577782
247.	15478	0	−2104649712	24293507361332
248.	15544	0	−1923770904	16032697721040
249.	15861	0	−2222843898	28223138924926
250.	16017	0	−1894608012	12669455257778
251.	16342	0	−2930687892	53360842513748
252.	16639	0	−2157138126	23937352680362
253.	16696	0	−1899342456	9734040486192
254.	17256	0	−1940191968	9944444590256
255.	17263	0	−1988780442	13469875004058
256.	17403	0	−2067010572	17561726654758
257.	17467	0	−2042256546	15956230911170
258.	17576	0	−1956479976	9464221804048
259.	17576	0	−2079882024	17692631650320
260.	18181	0	−2000399676	9724007315418
261.	18233	0	−2452133772	32966518760318
262.	18375	0	−2143257402	18541545743446
263.	18559	0	−2446738998	32137297657014
264.	18777	0	−2100220716	14434988174530
265.	18872	0	−2187177432	19343906493648
266.	19018	0	−3110703552	57142093712308
267.	19513	0	−2117832552	12253238564798
268.	19825	0	−2360618172	25518584902514
269.	20046	0	−2139520548	11144753720804
270.	20077	0	−2092659822	5180277482402
271.	20145	0	−2962543188	50121624626062
272.	20437	0	−2929953312	48449171525062
273.	20679	0	−4191210606	97444353070042
274.	21367	0	−2905391214	46108925570554
275.	21390	0	−2170947108	2284627162268
276.	21715	0	−3101610402	53511896831954
277.	21923	0	−2218288644	5508406033526
278.	21957	0	−2750882772	38629691251162
279.	22258	0	−2754883296	38233328638340
280.	22294	0	−2231281008	2182381970164

i	I	δ	p	q
281.	22317	0	−2315693622	13975399149118
282.	22328	0	−2310373032	13455069861552
283.	22601	0	−2889805248	43459627198306
284.	22638	0	−2636100192	31966003178692
285.	22659	0	−2536231764	26866023451850
286.	22762	0	−2349703224	14537286726644
287.	23768	0	−3025041624	47285015221136
288.	24023	0	−2391922566	11046420270490
289.	24389	0	−2373177906	4028862343682
290.	24389	0	−2394365688	8351153297894
291.	24389	0	−3879141684	81755014062810
292.	24513	0	−2400932424	8154710397394
293.	25161	0	−3884797488	81212487396194
294.	25257	0	−2450724474	8629909524214
295.	25838	0	−2620064436	21457997665500
296.	25913	0	−2592985032	19127543767618
297.	26243	0	−2969867472	40085925181962
298.	26299	0	−2552537328	13428724692966
299.	26494	0	−2868391548	34334014759484
300.	27178	0	−2812103712	29254059458932
301.	27334	0	−2579637588	8739003060084
302.	27397	0	−2558033412	1274312471718
303.	27542	0	−2610679896	11471880067340
304.	28189	0	−2843821266	27995128350222
305.	28393	0	−2777580234	22650082142666
306.	28902	0	−3903778164	77818087517236
307.	29167	0	−2890227384	27713966324594
308.	29310	0	−2675769492	1367689020676
309.	30122	0	−3075346032	36240318478452
310.	30759	0	−3285216264	46141853464190
311.	30777	0	−2820772812	14061117117058
312.	31051	0	−4314452406	93352535955822
313.	31179	0	−2926668756	22454459324858
314.	31949	0	−4151366538	85023772408014
315.	32322	0	−3319402152	44376123019228

i	I	δ	p	q
316.	33109	0	−3097415178	27986034000254
317.	33977	0	−3115972332	25895296767362
318.	34534	0	−3195647136	29954251797932
319.	36226	0	−4377249756	89957457427772
320.	37457	0	−3162345252	7274874836658
321.	38017	0	−4187659542	78152151927498
322.	38026	0	−3240158328	16295742104884
323.	40227	0	−3891789066	58223061420590
324.	40921	0	−3472772094	26002693453990

From the solutions of the index form equations we obtained 79 pairs of integral points on $y^2 = x^3 + \kappa$ as predicted by Broadhurst. Their coordinates are listed in the following table. The ordering is with respect to the size of the x-coordinate. The last column indicates the corresponding field F in the form $(f.i)$, f the conductor and i the number of the field in the tables of polynomials above.

	point $[x, \pm y]$	(f, i)
1.	$[938058, \pm 6852725]$	66.1
2.	$[944042, \pm 126272949]$	66.2
3.	$[945132, \pm 137340809]$	6.5
4.	$[952746, \pm 198573643]$	6.6
5.	$[971312, \pm 301631529]$	6.7
6.	$[1049742, \pm 575649251]$	6.8
7.	$[1067286, \pm 624775987]$	66.3
8.	$[1151051, \pm 836450142]$	33.1
9.	$[1161996, \pm 862303607]$	66.4

	point $[x, \pm y]$	(f, i)
10.	$[1178508, \pm 900781705]$	6.9
11.	$[1212671, \pm 978733332]$	1.7
12.	$[1372646, \pm 1326981843]$	22.4
13.	$[1378058, \pm 1338503925]$	66.5
14.	$[1409666, \pm 1405642053]$	6.10
15.	$[1426667, \pm 1441668426]$	33.2
16.	$[1521608, \pm 1642426665]$	66.6
17.	$[1568358, \pm 1741368365]$	66.7
18.	$[1668782, \pm 1954962909]$	2.5
19.	$[1707816, \pm 2038547753]$	66.8
20.	$[1951326, \pm 2569943933]$	6.11
21.	$[2184171, \pm 3097484918]$	33.3
22.	$[2203058, \pm 3141188325]$	22.6
23.	$[2300486, \pm 3368874387]$	6.12
24.	$[2990108, \pm 5090029065]$	22.8
25.	$[3523883, \pm 6552350970]$	33.4
26.	$[3726536, \pm 7136194263]$	66.9
27.	$[4167392, \pm 8458743849]$	6.13
28.	$[4500716, \pm 9504898953]$	66.10
29.	$[4562987, \pm 9704631354]$	33.5
30.	$[5375172, \pm 12428862281]$	66.11
31.	$[6449432, \pm 16353588009]$	66.12
32.	$[7217871, \pm 19370307232]$	3.17
33.	$[8823516, \pm 26193977353]$	6.14
34.	$[9298091, \pm 28337896422]$	11.1
35.	$[10730192, \pm 35137116201]$	66.13
36.	$[10941852, \pm 36182568439]$	6.15
37.	$[11134146, \pm 37141161157]$	6.16
38.	$[14123406, \pm 53069565923]$	66.14
39.	$[14513862, \pm 55286087021]$	66.15
40.	$[17039247, \pm 70329797144]$	3.3
41.	$[18003758, \pm 76386046755]$	6.17
42.	$[18723386, \pm 81011985813]$	22.12
43.	$[20267258, \pm 91236993045]$	66.16
44.	$[27514062, \pm 144318983779]$	6.18
45.	$[27602366, \pm 145014337347]$	66.17

	point $[x, \pm y]$	(f, i)
46.	$[29114336, \pm 157091639337]$	2.10
47.	$[33823652, \pm 196709853111]$	6.19
48.	$[35125508, \pm 208175580855]$	22.13
49.	$[37574262, \pm 230320349171]$	66.18
50.	$[39578922, \pm 248996390219]$	6.20
51.	$[89266452, \pm 843397206839]$	66.19
52.	$[92792472, \pm 893858743081]$	6.136
53.	$[98031002, \pm 970610471211]$	6.21
54.	$[101214542, \pm 1018272929949]$	22.15
55.	$[101792207, \pm 1027002795984]$	11.3
56.	$[105479807, \pm 1083312706116]$	3.81
57.	$[112809983, \pm 1198177543860]$	33.6
58.	$[129240906, \pm 1469264237173]$	6.114
59.	$[145164162, \pm 1748996975429]$	66.20
60.	$[181696383, \pm 2449172605220]$	33.7
61.	$[193717392, \pm 2696204912599]$	66.21
62.	$[217993611, \pm 3218589801262]$	33.8
63.	$[243970772, \pm 3810716920119]$	6.75
64.	$[394470102, \pm 7834677706061]$	6.144
65.	$[618995883, \pm 15400387195930]$	33.9
66.	$[723947306, \pm 19478720861877]$	6.123
67.	$[738487167, \pm 20068477632676]$	33.119
68.	$[761905551, \pm 21030602079608]$	33.98
69.	$[1944500046, \pm 85745618355043]$	6.147
70.	$[4000889882, \pm 253066639126059]$	22.121
71.	$[5581278396, \pm 416965889860343]$	6.151
72.	$[8039596136, \pm 720860699919063]$	66.204
73.	$[11398773672, \pm 1216990522012969]$	66.44
74.	$[13743516182, \pm 1611190109313741]$	66.283
75.	$[28046379108, \pm 4696942031534665]$	66.109
76.	$[31993006991, \pm 5722457703698472]$	33.127
77.	$[42649697387, \pm 8807931553841046]$	11.59
78.	$[242567289492, \pm 119467108642509001]$	66.198
79.	$[522048501292142, \pm 11927962909155860424099]$	22.89

Solving a "large" Thue equation

Performing the calculations for the last example with $\kappa = -825399814482565487$ we had to solve a huge number of Thue equations with unusually large coefficients and right hand sides. Most of them were solved directly by Magma with the command "IndexFormEquation". However, this was not possible for the following example.
For $(f, i) = (11, 6)$ we had to search for elements of index 6. One corresponding index form equation was

$$2535x^3 + 354x^2y - 81454xy^2 + 127855y^3 = \pm 6.$$

Multiplying it by 2535^2 and substituting $x_0 = 2535x$ and then $x_1 = x_0 + 118y$ we obtain

$$x_1^3 - 206527662x_1y^2 + 845993618459y^3 = \pm 2 \cdot 3^3 \cdot 5^2 \cdot 13^4. \qquad (1)$$

We had to solve this equation directly. Denote by K the number field generated over $\mathbb{Q}$ by ϑ with defining polynomial

$$f(x) = x^3 - 206527662x + 845993618459.$$

This is a totally real cubic field with integral basis

$$\left\{1, \vartheta, \frac{\vartheta^2 + 118\vartheta + 37}{2535}\right\}.$$

The fundamental units η_1, η_2 were calculated by Magma, they had coefficients of about 1600 digits in the above integral basis. Their conjugates were of magnitude up to 10^{2200}.
We found that in K there are
–two prime ideals over 2, of norm 2 and 4, respectively,
–two prime ideals over 3, each of norm 3,
–three prime ideals over 5, each of norm 5,
–three prime ideals over 13, each of norm 13.
Considering all possible products of these prime ideals we found that there are altogether 32 principal ideals of norm $2 \cdot 3^3 \cdot 5^2 \cdot 13^4$. Their conjugates were of magnitude up to 10^{2000}.
We then used standard arguments for solving equation (1). The challenge was to overcome the difficulties caused by the magnitudes of the units and the elements of given norm.
Equation (1) can be written as

$$N_{K/Q}(x_1 - \vartheta y) = \pm 38557350,$$

whence for any solution x_1, y of (1)

$$x_1 - \vartheta y = \gamma \cdot \eta_1^{a_1} \cdot \eta_2^{a_2} \tag{2}$$

where $\gamma \in \mathbb{Z}_K$ is of norm ± 38557350 and $a_1, a_2 \in \mathbb{Z}$. Set $A = \max(|a_1|, |a_2|)$. The usual arguments led us to linear forms in the logarithms of algebraic numbers of type

$$\Lambda = \left| \log \left| \frac{(\alpha^{(i)} - \alpha^{(j)})\gamma^{(k)}}{(\alpha^{(i)} - \alpha^{(k)})\gamma^{(j)}} \right| + a_1 \left| \frac{\eta_1^{(k)}}{\eta_1^{(j)}} \right| + a_2 \left| \frac{\eta_2^{(k)}}{\eta_2^{(j)}} \right| \right|$$

where $1 \leq i, j, k \leq 3$ are distinct indices. For non-zero y we had the upper estimate

$$\Lambda < 0.0004533 \cdot \exp(-7705.901 \cdot A + 46078.962) \tag{3}$$

and using the well known estimates of A.Baker and G.Wüstholz we obtained the lower estimate

$$\Lambda > \exp(5.62671 \cdot 10^{26} \cdot \log A).$$

This yields $A < 10^{25}$.

The reduction lemma of Baker and Davenport can be used to diminish this bound using the continued fraction algorithm. In one reduction step we got the reduced bound $A \leq 5$. The possible exponents a_1, a_2 below this bound were tested if there are corresponding x_1, y satisfying (2). There were no solutions, as we expected.

To perform this calculation we used Maple with 15000 digits precision for the reduction and 18000 digits precision for testing the possible small exponents. Surprisingly, with these extreme data and very high precision the standard algorithms worked well, the reduction was efficient and we got a final result. We had to perform the reduction procedure for all possible γ and for all possible i (for a given i the possible j and k play a symmetric role). The test of small exponents was performed for all possible γ and for $-5 \leq a_1, a_2 \leq 5$. The CPU time on an average laptop took about 10 minutes.